THE METHOD OF SOLVING OF NON-STATIONARY COUPLED PROBLEMS OF THE THEORY THERMAL-PLASTICITY FOR THE ROTATION SHELLS

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Summary The new on-component splitting method of higher accuracy of numerical solving of non-stationary coupled problems of the theory of thermal-plasticity for non-steady loading of the rotation shells is proposed. The numerical solving is reduced to the solution of the systems of differential equation in partial derivatives, consisting of equations of motion, geometrical ratios, equations of physic state and equation of thermal conduction. All unknown values for each step by time are found in the spline-function form.

INTRODUCTION

In the papers [1,2] the method for on-component splitting of higher accuracy is proposed. The space non-stationary problems of theory of thermal-plasticity are solved by this method [7]. All unknown values (velocity of displacements, strains and deformations) are found in form of spline functions (cubic B-splines, strained splines) [1-3]. The application of this method for the solution of non-stationary problems of thermal-plasticity for plates and shells is described in papers [4,5]. The non-stationary two-dimensional and three-dimensional coupled problems of the theory of thermal-plasticity are studied in paper [6]. In present paper this method is applied for the first time for solving of coupled non-stationary problems of the theory of thermal-plasticity for rotation shells. The method proposed by us allows the fourth order of approximation of the method on coordinates.

THE PROBLEM

In the present paper the above-mentioned technique is used for determination of coupled non-stationary thermal-strains-deforming state of thin-walled rotation shells working beyond the limits of the material elasticity. The basic unknown values are velocities of displacements V_1, V_2, V_3, specific efforts Q_1, Q_2, , T_1, T_2, S, specific moments M_1, M_2, H, deformation of a median surface ε_1 ε_2, shifts γ_1, γ_2, angles of rotation ω_1, ω_2, curvatures χ_1, χ_2, torsion τ and temperature T.

The complete set of simultaneous equations consists of:

- the equations of motion of thin-walled element of construction;
- the equations for speeds of the specific efforts and moments change which give the physical ratio (we use the ratio of theory of processes of deformations of mean curvature elaborated by the Academician Y.N.Shevchenko and his disciples [7]);
- the nonlinear geometrical ratio.

The temperature field is constructed by solution of non-stationary two-dimensional equation of heat conduction with account of probable influence of heat emission in the process of loading of rotation shells [3].

The above mentioned simultaneous equations may be reduced to vector form

$$\frac{\partial \vec{W}}{\partial t} = \sum_{i=1}^{2}\left(A_i \frac{\partial \vec{W}}{\partial \alpha_i} \right) + \vec{F} \quad , \tag{1}$$

to which the method of on-component slitting where the unknown values are given in the spline-function form (cubic B-splines and strained splines) may be applied [1].

In general vector $\vec{W}$ has 21 components: $w_1=v_1$; $w_2=v_2$; $w_3=v_3$; $w_4=Q_1$; $w_5=Q_2$; $w_6=T_1$; $w_7=T_2$; $w_8=S$; $w_9=M_1$; $w_{10}=M_2$; $w_{11}=H$; $w_{12}=\varepsilon_1$; $w_{13}=\varepsilon_2$; $w_{14}=\gamma_1$; $w_{15}=\gamma_2$; $w_{16}=\omega_1$; $w_{17}=\omega_2$; $w_{18}=\chi_1$; $w_{19}=\chi_2$; $w_{20}=\tau$; $w_{21}=T$.

The 20 components of vector $\vec{F}$ $(F_i, i=1,2,...,20)$ are defined by the expression introduced in paper [5].

For the solution of bound non-stationary problems for F_{21} we use the expression [3]

$$F_{21} = S_{ij}\dot{Э}_{ij} - \frac{1}{2G} S_{ij}\dot{S}_{ij} + \frac{\sigma_{ii}}{3}\left(\dot{\varepsilon}_{jj} - 3\alpha_T \frac{\partial T}{\partial t} \right) - \frac{\sigma_{ii}}{3K}\dot{\sigma}_{jj} \, , \quad S_{ij} = \sigma_{ij} - \delta_{ij}\sigma, \; Э_{ij} = \varepsilon_{ij} - \delta_{ij}\varepsilon \, .$$

NUMERICAL METHOD

The non-explicit form of the on-component splitting method scheme for equation (1) has form ($\alpha+\beta=1$, $\phi_1+\phi_2=1$, $n=1,2$)

$$\vec{W}^{p+\frac{n}{3}} - \alpha\tau\left[A_n\lambda_n(\vec{W})\right]^{p+\frac{n}{3}} = \vec{R}_n\,,\ \vec{R}_n = \vec{W}^{p+\frac{n-1}{3}} + \beta\tau\left[A_n\lambda_n(\vec{W})\right]^{p+\frac{n-1}{3}} + \tau\phi_n\vec{F}\,. \tag{2}$$

Let us rewrite the equation (2) in the form:

$$\vec{W}_j^{p+\frac{n}{3}} = \alpha\tau\left[A_n\lambda_n(\vec{W})\right]_{j-1}^{p+\frac{n}{3}} = \vec{R}_n\,, \tag{3}$$

λ_n - the difference operator [1], j – number of iteration *(j= 1, 2, 3…)*. The initial iteration is the solution of the problem obtained by the explicit scheme. Thus,

$$\vec{W}_0^{p+\frac{n}{3}} = \left\{\vec{W} + \tau\left[\phi_n\vec{F} + A_n\lambda_n(\vec{W})\right]\right\}^{p+\frac{n}{3}}. \tag{4}$$

If $\max(W_m;\lambda_n(W_m)) < \Lambda$, then $(W_m)_{i+1} - (W_m)_i < (\alpha\frac{\tilde{\tau}}{h_k}\Lambda)^i$, $(k=1,2)$ and, therefore at $\alpha\tilde{\tau}\Lambda < h_k$ the iterative procedure converges.

NUMERICAL RESULTS

We consider the problem of non-stationary loading of previously contracted cylindrical shell ($T_1 = T_{10}$-const) subjected to the cyclic torsion of small amplitude, when the non-zero value of the normal velocity of displacement of a medial surface of a cylindrical shell (V_3 =0,05 and V_3 =0,1) is given for initial time moment. Numerical values of dimensionless velocities of radial displacement for different points in time are given in Table 1.

Table 1.

V_3=V_3(t)	$t = 25\tilde{\tau}$	$t = 50\tilde{\tau}$	$t = 75\tilde{\tau}$	$t = 100\tilde{\tau}$
V_3(0)=0,05; T=0	-0,021	0,012	0,043	-0,026
V_3(0)=0,05; T=T(t)	-0,005	-0,007	0,009	0,008
V_3(0)=0,1; T=0	-0,045	0,024	0,089	-0,044
V_3(0)=0,1; T=T(t)	-0,012	-0,016	0,023	0,018

lines 3 and 5 are constructed by the on-component splitting method, and lines 2 and 4 represent similar results obtained for the case when temperature equals zero.

The received results show that the emission of heat in the process of cyclic loading beyond the material elasticity limit may considerably influence the strain-deformed state of the cylindrical shell.

CONCLUSIONS

Thus, the on-component splitting method of higher accuracy for solving of non-stationary coupled problems of the theory of thermal-plasticity for rotation shells proposed by us allows to determine the unknown values of velocities of displacements of the medial surface of the shell, specific efforts and specific moments, deformation of the medial surface, shifts, angles of rotation, curvatures, torsion and temperature. The absence of the derivatives of the second order on coordinates in the complete set of simultaneous equations gives the opportunity to get the fourth order of approximation of the method on coordinates which results in higher accuracy of calculations.

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