MODEL UPDATING A MULTICRITERIA OPTIMIZATION PROCESS IN MECHANICS

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Summary Model updating is a widely used area of research in the field of dynamics of elasto-mechanics. The ill-conditioned inverse mathematical problem is mainly solved in a sensitivity based numerical process, considering regularization methods. Here it is shown how additional dynamical properties of the structure can be introduced in the updating procedure. Model updating will be formulated as a constrained multiobjective optimization problem, which can be solved by a hierarchical scalarization method. Applications will be shown.

INTRODUCTION

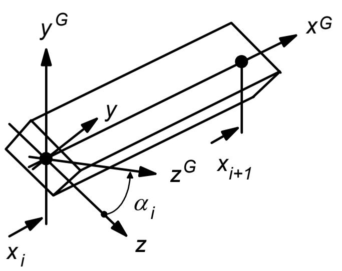
Model updating is the tool for achieving a validated mathematical model of an elasto-mechanical system. The purpose of model updating is to converge the mathematical model of a system upon measurements that are taken from the real structure. Thinking in input/output relations, the goal is to adjust the model parameters by comparing measured dynamical i/o relations of the system with numerical i/o relations of the mathematical model.
Model updating has become a very wide-spread area of research in the field of FE-modelling of structures. The Finite Element method is the tool most used for numerical modelling in structural engineering today. But for many applications, an initial FE-model is often a poor representation of a real structure because of simplifying and idealization while constructing the FE-model. A special issue of the Journal of *Mechanical System and Signal Processing* [1] demonstrated the developments in model updating. Typical examples and benchmark tests include the recently completed COST action [2] (European CO-operation in the field of Scientific and Technical research). Updated and validated mathematical models are of very great importance in the area of industrial developments of mechatronic systems, in the field of optimization of dynamic systems, for structural modification problems and for model based damage detection.
The common way of model updating is based on sensitivity methods. Here, an error function is defined as the difference between the numerical and the measured data of a dynamic system. The errors are defined in the modal domain (natural frequencies and mode shapes) and in the frequency domain (FRFs: frequency response functions). The model updating problem is solved by minimizing a global function of all system errors in an iterative numerical process. Usually this inverse mathematical problem is ill-conditioned. For its solution it is necessary to apply "regularization" methods by using additional constraints.

MODEL UPDATING DESCRIBED BY FUNDAMENTALS OF MECHANICS

Model updating is an ill-conditioned inverse mathematical problem; adjusting of model parameters has to be calculated from the output of the model. Facing this fact, it is necessary to go back to the fundamentals of mechanics. Instead of applying formal regularization methods, we have to deal with invariants of the elasto-mechanical system and with the application of the principles of mechanics.
The goal is the generation of a mathematical model, which is substantially condensed and at the same time highly accurate, regarding dynamical system properties.

Description of the elasto-mechanical structure using FE-super-elements

The updated model is a highly accurate dynamic description of the real system using FE-macro-elements, e. g. macro-beam-elements (see Fig. 1). Due to application in system modification and optimization and model based damage detection, its layout has to be close to reality. It is described by design variables x_i, $i = 1,\dots, n$. These are: element masses, positions of the element nodes, angles of principal axes of the cross sections and parameters of bending, torsion and axial stiffness. The model will be described by the vector $\boldsymbol{x} = [x_1, \dots, x_n]^T$ of n design variables. Finally, this design vector is the result of the updating process.



parameters of an element i	
coordinates of element nodes	x_i
	x_{i+1}
mass	m_i
stiffness parameters	EA_i
	EI_{yyi}
	EI_{zzi}
	GI_{pxi}
principal axis	α_i

Fig. 1: Parameters of FE-macro-element (y^G, z^G: global coordinates)

Considered dynamical properties of an elasto-mechanical structure

Regarding the fundamentals of mechanics, model updating means the adaptation of all mechanical invariants and dynamical properties to the mathematical model. Considering more of such properties is equal of making the ill-

Type	Dynamical property	
Mass-geometrical data	Total mass	$m = \sum_{i=1}^{p} m_i$
	Centre of mass	$\boldsymbol{r}_{OS} = [x_{OS}, y_{OS}, z_{OS}]^T$
	Tensor of inertia	$\boldsymbol{I}_S = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ & I_{yy} & I_{yz} \\ sym. & & I_{zz} \end{bmatrix}$
Modal Data	Nat. frequencies	$\omega_i\,,\ i = 1, \dots ,k$
	Nat. mode shape	$\boldsymbol{q}_i\,,\ i = 1, \dots ,k$
Frequency Response function	Selected FRF-data	$h_j(\omega_i)\,,\ i = 1, \dots ,k,\ j = 1, \dots ,p$

Table 1: Dynamical properties for model updating process

conditioned inverse mathematical problem less under-determinated. If we are considering only modal values and FRF-data, that means simply that we are dealing with mathematical models, which are far away from the reality of mechanics. Regularization methods cannot close this gap. They make sense only in numerical adjusting, not in better mechanical modelling.
A reasonable model updating process has to consider at least all mass-geometrical values of the real elasto-mechanical structure, all modal values in a concerned frequency range (natural frequencies f_i and natural mode shapes $\boldsymbol{q}_i$) and all selected FRF-data near to the natural frequencies f_i (see table 1). The updated model will be correct in a mechanical sense, only if the time-variable forces inside of the structure are correctly updated to a state of a vibration. We can construct a vector of *local momentum*, which is composed of the linear momentums of each macro-element of the system in a state of a vibration [3]. Using this vector in the updating procedure means we additionally consider the internal forces of the structure.

MODEL UPDATING: A MULTICRITERIA OPTIMIZATION PROBLEM

All dynamical properties can be formulated as functions of the design vector $\boldsymbol{x}$. The error expressions $\varepsilon_k(\boldsymbol{x})$ between the numerical and measured system properties are elements of the vector objective function $\boldsymbol{f}[\varepsilon(\boldsymbol{x})]$ (1). Finally, the model updating procedure can be formulated as a constrained multiobjective optimization problem (2) Where $\boldsymbol{h}(\boldsymbol{x})$ = equation of constraints; $\boldsymbol{x}_L$ and $\boldsymbol{x}_U$ are the lower and upper bounds, respectively.

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} \varepsilon_1(\boldsymbol{x}) \\ \varepsilon_2(\boldsymbol{x}) \\ \varepsilon_3(\boldsymbol{x}) \\ \varepsilon_4(\boldsymbol{x}) \\ \varepsilon_5(\boldsymbol{x}) \\ \varepsilon_6(\boldsymbol{x}) \end{bmatrix} = \begin{bmatrix} \text{error expression of: total mass} \\ \text{... position of centre of mass} \\ \text{... elements of tensor of inertia} \\ \text{... natural frequencies } f_1(\boldsymbol{x}) \dots f_k(\boldsymbol{x}) \\ \text{... natural mode shapes } \boldsymbol{q}_1(\boldsymbol{x}) \dots \boldsymbol{q}_k(\boldsymbol{x}) \\ \text{... selected FRF - datas} \end{bmatrix}, \quad (1)$$

$$\min_{\boldsymbol{x} \in \Sigma} \left\{ \boldsymbol{f}\left[\varepsilon(\boldsymbol{x})\right] \middle| \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0} \right\}, \quad \Sigma := \left\{ \boldsymbol{x} \in \Re^n \middle| \boldsymbol{x}_L \le \boldsymbol{x} \le \boldsymbol{x}_U \right\}. \quad (2)$$

This optimization problem can be solved numerically by a hierarchical scalarization strategy in consecutive steps using a SQP algorithm. The elements of the vector objective function (1) are divided in three different groups by scalarization (3): *1st group*: all mass-geometrical values; *2nd group*: the considered eigenfrequencies and mode shapes; *3rd group*: the considered FRF-values. In each step of the hierarchical optimization procedure, one group of dynamical properties of the mathematical model will be adapted to the properties of the real system. Those system properties, which already have been adapted in previous steps of the procedure, must be fixed by additional constraints in the following steps. The final vector $\boldsymbol{x}$ of the design variables creates the updated model for the elasto-mechanical structure considered here. The great advantage of this formulation is the fact that each different system property can be updated in a special process, whereas in sensitivity based methods all errors between model and measurements are regarded simultaneously. This model updating procedure will be shown for several elasto-mechanical structures.

$$\boldsymbol{f}(\boldsymbol{x}) \quad \Rightarrow \quad \boldsymbol{s}\left\{ \boldsymbol{f}\left[\varepsilon(\boldsymbol{x})\right] \right\} = \begin{bmatrix} s_1\left[\varepsilon_1(\boldsymbol{x}), \varepsilon_2(\boldsymbol{x}), \varepsilon_3(\boldsymbol{x})\right] \\ s_2\left[\varepsilon_4(\boldsymbol{x})\right] \\ s_3\left[\varepsilon_5(\boldsymbol{x})\right] \end{bmatrix}. \quad (3)$$

CONCLUSIONS

The creation of updated models can be solved in three main steps. First, the elasto-mechanical system has to be constructed by FE-macro-elements whose lay-out is close to reality. All system parameters are collected in a design vector $\boldsymbol{x}$. Next, a number of dynamic system properties form the basis of the updating procedure. Finally, this process can be formulated as a constrained multiobjective optimization problem, where a vector objective function of system errors will be minimized. A special hierarchical scalarization strategy is shown for the solution of this problem.

References

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