RAPID FORMATION OF STRONG GRADIENTS AND DIFFUSION IN THE TRANSPORT OF SCALAR AND VECTOR FIELDS

Konrad Bajer
Institute of Geophysics, Warsaw University, ul. Pasteura 7, 02-093 Warszawa, Poland

Summary An important issue in the theory of transport by moving fluids is the role of dissipation when the medium is *nearly ideal*. The central problem of this nature is understanding viscous dissipation at very large Reynolds numbers. We will discuss a few problems in the same category but *linear* and therefore more promising although, as it turns out, surprisingly rich and far from being resolved. Their common denominator is the interplay between diffusion and advection. In a typical flow the latter tends to decrease the characteristic length scales of the spatial variations of the transported quantity, thus increasing the rate of diffusion. Depending on a particular configuration either this rapid diffusion prevails and efficiently annihilates all gradients, or a kind of balance is reached and a quasi-steady dissipative structure emerges.

ACCELERATED DIFFUSION INDUCED BY VORTICES

Concentrated vortex has strong influence on the diffusion of passive scalars and vectors, like magnetic fields, in its neighbourhood [1,3-9,12-17]. Placed in a region of the large-scale gradient of an advected quantity, say temperature, it wraps the isotherms into spirals and therefore decreases the radial length-scale. This causes the process known as *accelerated diffusion*. The gradients of, say, temperature are expelled from the vicinity of the vortex which is then surrounded by a nearly isothermal region.

In two-dimensional magnetohydrodynamics this process, called *flux expulsion*, was noticed quite early [17], but initially there was an uncertainty as to the time-scale on which it occured. Much later simple models and arguments were put forward [12,16] proving this process to be quite rapid. It is faster by the factor $Pe^{-2/3}$ than normal diffusion across the vortex. Here $Pe = \Gamma/2\pi\kappa$ is the appropriate dimensionless number based on the total circulation Γ of the vortex and measuring the relative magnitudes of the advective and diffusive terms. In the case of temperature κ is thermal diffusivity and Pe is the Péclet number. Their MHD analogues are magnetic diffusivity and magnetic Reynolds number respectively.

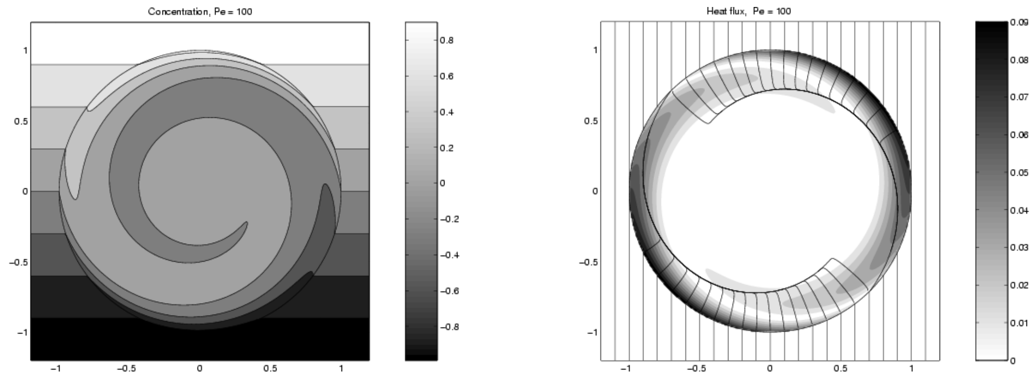

Figure 1. Cylindrical cavity filled with inviscid fluid surrounded by solid with imposed uniform temperature gradient. The steady flow in the cavity is due to a point vortex at the centre. In a fluid with finite thermal diffusivity a non-trivial steady state is reached with uniform temperature around the vortex and spiral isotherms in a thermal boundary layer near the wall (left panel). The right panel shows the integral lines of the heat flux which is completely expelled from the central area. The figure shows an exact analytical solution with $Pe = 100$. From [1].

The evolution equations of the mechanical quantities in fluids, like momentum and its derivatives, also contain advective terms and Fickian diffusion and therefore are also subject to accelerated diffusion. Batchelor [7] demonstrated the tendency of vorticity to become uniform but the mechanism and the time-scale were still unknown.

If a strong streamwise vortex is embedded in a boundary layer, then the streamwise component of *velocity* of the boundary layer flow is governed by the same advection-diffusion equation as for passive scalar [13-15]. The accelerated diffusion of streamwise momentum locally induces a complex velocity profile in the boundary layer. Due to the rapidity of the process it may have signifficant impact even if the streamwise vortex 'lives' only for a few turnover times.

STRONG COHERENT VORTEX INTERACTING WITH WEAK AMBIENT VORTICITY

When a strong concentrated vortex is embedded in the 'sea' of background vorticity aligned with it but *weak*, then this background (possibly a small perturbation of the strong vortex) is governed by an equation similar but not the same as the 'normal' advection-diffusion [4]. However, accelerated diffusion still operates creating spiral structures in the background vorticity. Those structures may in turn affect the vortex which is then displaced from its initial position. To illustrate this, we show, theoretically and numerically, a self-induced motion of a single vortex in a Poiseuille flow. Modern experiments on turbulence in liquids as well as numerical simulations reveal the presence of elongated vortices whose diameter is somewhere between the Kolmogorov and the Taylor microscale. At small scales vorticity seems to be organised in a web of filaments and weak background. We will discuss the interaction between them.

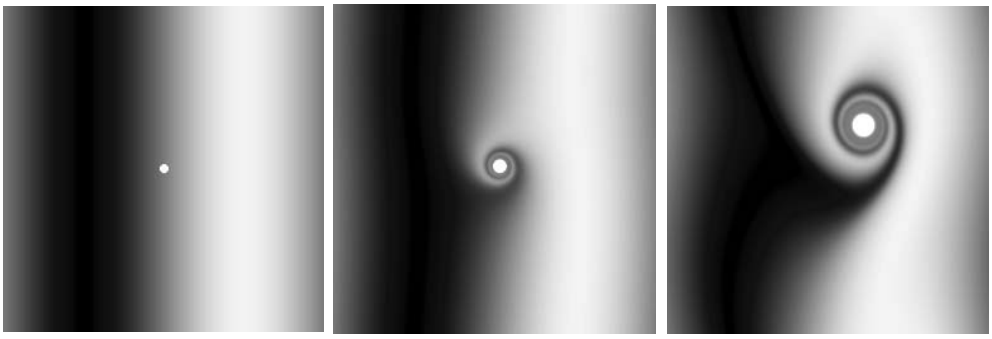

Figure 2. Strong gaussian vortex started in a weak background vorticity. The vortex creates a perturbation by winding non-uniform background into spirals. The flow associated with this perturbation displaces the vortex from its initial position. From [4].

FORMATION OF CURRENT SHEETS IN MAGNETOHYDRODYNAMICS

Vortex filaments are in many ways analogous to the magnetic flux tubes which are coherent structures found in magnetohydrodynamics (MHD), especially prominent in the solar dynamo process. Interacting flux tubes create current sheets and, in the ideal limit, tangential discontinuities - the generic MHD singularities which form in the process of relaxation towards magnetostatic equilibrium [10]. The conditions in the solar photosphere are nearly ideal, so the topological constraints imposed by non-dissipative MHD are important, as are the singularities where these constraints are most easily broken.

Both slender vortices and flux tubes can have topologically complex form (e.g. knotted or linked) and the mathematical apparatus necessary to describe and classify the topology is the same. Their steady states are mathematically equivalent, but there are important differences in their evolution [11]. The influence of the topology on the dynamics provides an important common ground. We will explore different aspects of the formation of strong gradients in vortex dynamics and in magnetohydrodynamics exploiting the analogies between the the governing equations.

Current sheets are believed to play an important dynamical role being a seat of vigorous Ohmic heating. However, the rate of Ohmic heating in a *single* sheet in a quasi-steady state *decreases* with increasing electrical conductivity of the fluid. Therefore, such individual sheets cannot provide efficient source of heat in a highly conducting medium like, for example, plasma in the solar corona. Qualitative models, usually involving dissipation related to turbulence driven by magnetic reconnection in the sheet, are often invoked to explain this apparent heating deficit. We will discuss a possibility of increased heat release in a *set* of simultaneous current sheets that are likely to form in any relaxing magnetic field that has *chaotic field lines* [2].

The relaxation process in a perfectly conducting fluid preserves all topological invariants. This set of constraints imposes strict lower bounds on magnetic energy of the equilibrium state. With reconnection, at least some of the constraints are relaxed, the topology of the field lines may change and lower energy states become accessible. If a multi-scale system of current sheets is present than we may expect less energetic final equilibria and more dissipation in the meantime.

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