PROBABILITY DISTRIBUTION FUNCTIONS FOR A RAPIDLY ROTATING TURBULENT FLOW: EXPERIMENT AND THEORY

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We consider a turbulent flow that rotates sufficiently fast so that the Rossby number is small (0.1), that is, the Coriolis force dominates inertial effects. Further, the dissipation, which is due mainly to Ekman friction in the boundary layers, is small. Hence the flow is essentially 2-dimensional (2D), like flows in the atmosphere and oceans on large spatial scales. Such a flow is different from the non-rotating quasi-2D turbulent flows studied in soap films [1] and thin electrolyte layers [2], and different from strictly 2D turbulent flows, which do not exist in nature but have been studied theoretically and numerically. Our study concerns the statistical properties of the low Rossby number flow: the probability distribution function (PDF) measured for the vorticity ω (the component parallel to the rotation axis), and the PDF of increments of a component of the velocity, $\delta v(x) = v(r+x)-v(r)$, where v lies along the line connecting r and $r+x$ [3,4].

Our apparatus is an annular tank with inner diameter 21.6 cm and outer diameter 86.4 cm. The bottom slopes to mimic the earth's beta effect: the depth increases from 17.1 cm at the inner radius to 20.3 cm at the outer radius. Fluid is pumped into the tank through a ring of 120 holes in the tank bottom at a radius of 18.9 cm, and out of the tank through a ring of 120 holes at a radius of 35.1 cm. The result of this radial forcing is a broad azimuthal counter-rotating jet that is turbulent (Reynolds number 20000) [4]. Measurements of the azimuthal component of the velocity are made with hot film probes place midway between the inner and outer radii of the annulus, and these velocity time series measurements are complemented by Particle Image Velocity measurements of the entire 2D velocity field.

The velocity measurements yield a PDF for the velocity increments that is non-Gaussian, but the functional form for $P(\delta v)$ is the same for different distances x, as Fig. 1 illustrates. A consequence of this *self-similar* behavior is that the exponents $\zeta(p)$ for the structure function,

$$S_p(x) = \langle (\delta v)^p \rangle \sim x^{\zeta(p)},$$

should be linear in the order p. We have confirmed this linearity in direct determinations of the scaling of S_p for orders up to $p = 10$ [4].

Equilibrium statistical mechanics has long been used to gain insight into turbulence (e.g., [5-6]). The analyses are usually based on Boltzmann-Gibbs statistical mechanics, which describes weak interactions and does not capture the long range interactions present in low Rossby number flows where there are large coherent jets and vortices. We explore the applicability of a generalization of statistical mechanics proposed by Tsallis, known as nonextensive statistical mechanics [7]. Energy and enstrophy are both approximately conserved in weakly dissipative low Rossby number flows. Assuming that these quantities are conserved in our flow, we derive a form for the PDF of the vorticity by maximizing a nonextensive entropy function. In order to apply the formalism to our experiment, we had to reduce the number of boundary conditions, which was done by pumping fluid through a *single* ring with a semi-circle of 60 holes that were sources, and on the opposite side of this circle was a semi-circle of 60 sinks. The flow configuration is described in [8]. The resultant turbulent flow is turbulent for the pumping rate used [8]. We find that $P(\omega)$ for this flow, computed from our nonextensive analysis, is in good accord with the observations, as Fig. 2 demonstrates.

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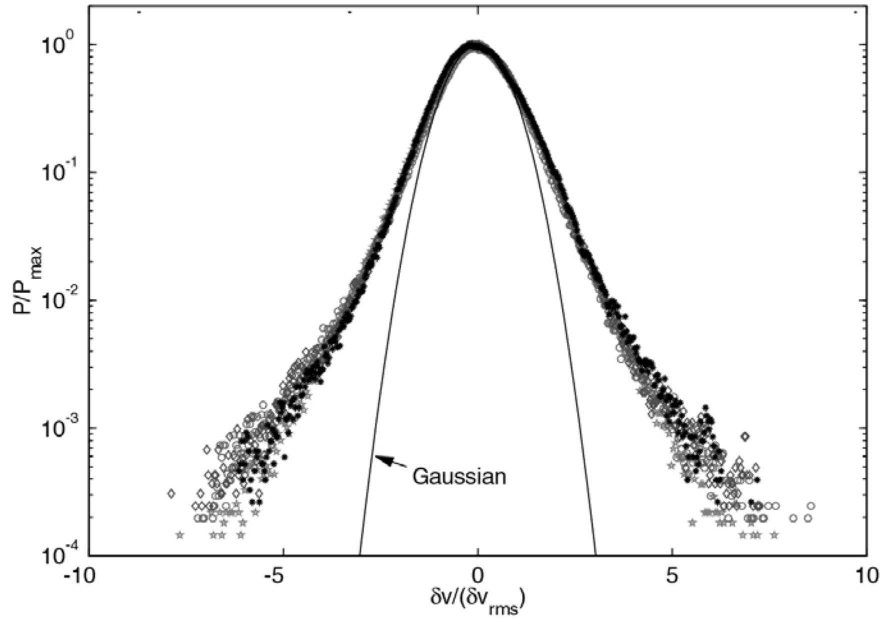

Fig. 1. The velocity increment PDF is *self-similar*, i.e., the functional form of *P(δv)* (normalized by its maximum value) is independent of the distance between measurement points. Data for six distances *x* ranging from 1.1 to 17.3 cm are shown by different symbols (for comparison, the Kolmogorov dissipation length is 0.07 cm) [4]. The width of the PDF for each distance *x* is scaled by the corresponding rms value of δv. (from [4]).

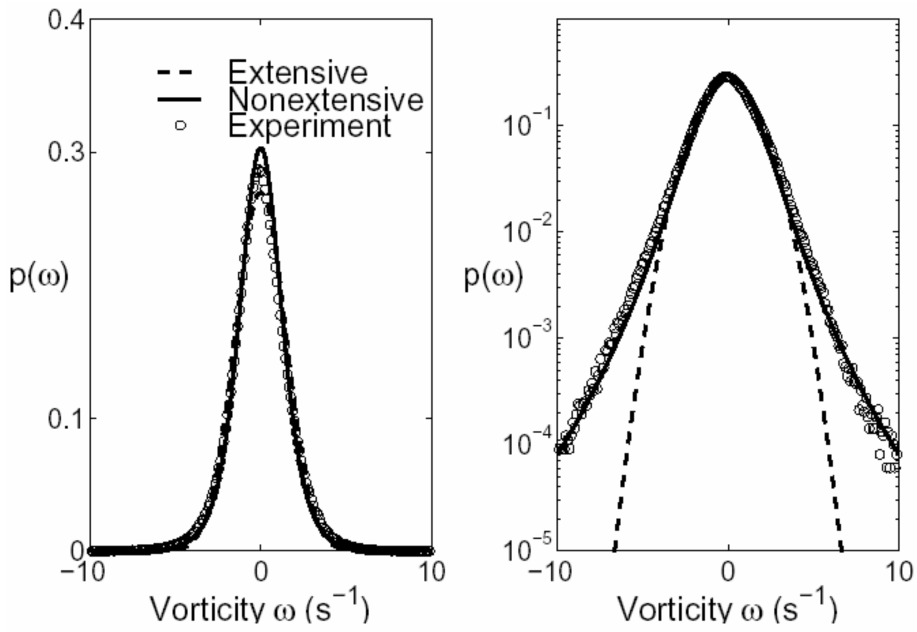

Fig. 2. Vorticity PDF on linear (left) and log (right) scales: comparison of laboratory measurements (o) with predictions obtained from statistical theory. The prediction from nonextensive statistical mechanics agrees well with the observations, while the Gaussian curve given by Boltzmann-Gibbs statistical mechanics is far from the observations. (from [3])