

TURBULENCE AND LARGE-EDDY SIMULATIONS

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Summary After having discussed the limits of turbulence direct-numerical simulations, one presents large-eddy simulations methods, where small scales are filtered out and modelled by appropriate eddy coefficients in the evolution of large scales. We concentrate on models developed originally in Fourier space. One presents coherent-vortex dynamics obtained thanks to these models for incompressible isotropic turbulence and channel flow at low Mach. Then the controlled compressible jet (Reynolds 36000) is studied at Mach 0.7 and 1.4.

THE NEED FOR LES

Direct-numerical simulations (DNS) of turbulence are based on deterministic solutions of Navier-Stokes equation, obtained through a proper discretization on a spatio-temporal grid of partial-differential operators. This implies that the typical grid mesh Δx in space should be inferior to the Kolmogorov scale l_D , under which velocity fluctuations are damped by molecular viscosity. One can thus show that the number of spatial grid points necessary for a well-resolved DNS is $\sim R_\lambda^{9/2}$, where $R_\lambda = u'\lambda/\nu$ is defined thanks to the rms longitudinal velocity and the Taylor microscale. R_λ may be determined experimentally, for instance on a commercial-plane wing where it is equal to 3000 (Jimenez [1]). This yields $\approx 10^{15}$ grid points to handle on the computer, which permits to envisage a DNS of such a flow within 30 to 50 years. For the atmospheric boundary layer it is worst, since we have $R_\lambda = 10000$, and hence 10^{18} grid points. To be able to perform a simulation in such cases, one is thus obliged to reduce drastically the number of degrees of freedom of the system. Large-eddy simulations (LES) are a powerful tool for this purpose. More details concerning the rest of this talk may be found in Lesieur [2] and Lesieur & Métais [3].

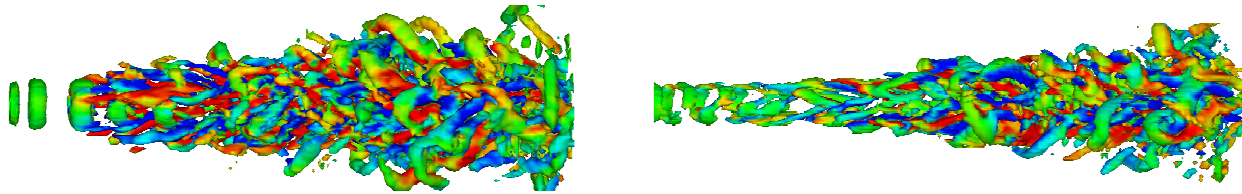
INCOMPRESSIBLE LES

Physical space

Density ρ is uniform. Let Δx be a given spatial grid mesh, and $G_{\Delta x}$ a low-pass filter of width Δx , chosen in order to eliminate subgrid-scale motions of wavelength $< \Delta x$. One defines for any quantity $\bar{f}(\vec{x}, t) = f * G_{\Delta x}$, where $*$ stands for a convolution product. The filter commutes with spatial and temporal derivatives (if Δx is uniform). When it is applied to Navier-Stokes equation, one obtains the same equation for the filtered velocity $\bar{u}_i$, provided a subgrid-stress tensor $T_{ij} = \bar{u}_i \bar{u}_j - \overline{u_i u_j}$ is added to the filtered viscous stress $2\nu \bar{S}_{ij}$. Here S_{ij} is the deformation tensor, symmetric part of the velocity gradient. An eddy-viscosity assumption $T_{ij} = 2\nu_t(\vec{x}, t) \bar{S}_{ij} + (1/3) T_{ll} \delta_{ij}$ allows to form for $\bar{u}_i$ a Navier-Stokes equation where pressure has been replaced by a macro-pressure $\bar{P} = \bar{p} - (1/3)\rho T_{ll}$. The same procedure applied to a scalar $T(\vec{x}, t)$ transported by the flow (with a molecular diffusivity κ) yields for $\bar{T}$ a similar equation, with an extra eddy diffusivity κ_t , given from ν_t thanks to a turbulent “Prandtl number” $P_r^{(t)} = \nu_t/\kappa_t$. These eddy coefficients need to be determined. In Smagorinsky’s model, the eddy viscosity is based on velocity gradients and taken proportional to $(\Delta x)^2 \sqrt{\bar{S}_{ij} \bar{S}_{ij}}$. There are interesting improvements of this model made by Germano et al. [4] where the constant is calculated dynamically by a double filtering. In fact, the major drawback of an eddy-viscosity assumption in physical space is that it assumes a scale-separation between filtered and subgrid-scales. This explains the poor correlation between T_{ij} and $\bar{S}_{ij}$ found in a-priori tests based on DNS. This is a serious motivation to work in Fourier space if the geometry of the problem permits it.

Fourier space

Turbulence is first assumed statistically homogeneous. Let $\hat{f}(\vec{k}, t)$ be the spatial Fourier transform of any quantity $f(\vec{x}, t)$. The filter is a sharp filter, such that $\bar{\hat{f}} = \hat{f}$ for $k = |\vec{k}| < k_C = \pi/\Delta x$; $\bar{\hat{f}} = 0$ for $k > k_C$. If turbulence is statistically isotropic, one can define the kinetic-energy spectrum $E(k, t)$, mean kinetic energy per unit mass at the wavenumber k . In Fourier space, nonlinear interactions go through “resonant” triads of wavevectors such that $\vec{k} = \vec{p} + \vec{q}$. The subgrid modelling consists here in evaluating the momentum transfers due to triads where $k < k_C$ and one at least of wavenumbers p and q is larger than k_C . Since the Fourier transform of Navier-Stokes dissipative term is $-\nu k^2 \hat{u}_i(\vec{k}, t)$, the subgrid momentum transfer will be modelled as $-\nu_t(k|k_C) k^2 \hat{u}_i(\vec{k}, t)$, the eddy viscosity in spectral space being calculated at the level of subgrid kinetic-energy transfers through an advanced theory of turbulence, the EDQNM. One gets $\nu_t(k|k_C) = 0.44 C_K^{-3/2} [E(k_C)/k_C]^{1/2} X(k/k_C)$, assuming that k_C lies in a Kolmogorov spectrum $E(k) =$



$C_K \epsilon^{2/3} k^{-5/3}$. Here, $X(k/k_C)$ is a “plateau-peak function” equal to 1 for $k/k_C < \approx 1/3$, and rising above. In the spectral-dynamic model, one accounts for a spectral slope at k_C different from $5/3$. We give two applications of this model : for isotropic turbulence at infinite Reynolds number, one presents an animation showing the formation and evolution of the spaghetti-type vortices, seen thanks to the vorticity and the second invariant of the velocity gradient $Q = (1/2)(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij})$. The spectral-dynamic model has also been applied to a channel at $h^+ = hv_*/\nu = 204$ and 395 . It yields very good statistical results compared with Kim’s DNS. At $h^+ = 395$, the LES is 70 times faster than the DNS.

Return to physical space : structure-function models

For complicated geometries, numerical methods impose to work in physical space. The spectral eddy viscosity is thus set to a constant calculated assuming the subgrid kinetic-energy dissipation equals ϵ in a Kolmogorov cascade. We have $\nu_t(\vec{x}, \Delta x) = (2/3)C_K^{-3/2} [E_{\vec{x}}(k_C)/k_C]^{1/2}$, where $E_{\vec{x}}(k_C)$ is a local kinetic-energy spectrum, determined with the aid of the local second-order velocity structure function. This model improves classical Smagorinsky model for non-sheared turbulence. For sheared turbulence (without or with walls), two excellent versions of the model exist to eliminate the damping effects of large-scale shears : the selective structure-function model, and the filtered structure-function model. They work very well for free-shear flows and boundary layers (in the incompressible and compressible cases), and can be utilized on unstructured grids.

COMPRESSIBLE LES

We work with an ideal gas. The LES formalism is much more complicated in the compressible case. One can simplify it a lot by using Favre filtering $\tilde{f}$, and introducing a “macro-temperature” $\vartheta = \tilde{T} - \mathcal{T}_{ll}/(2C_v\bar{\rho})$, where $\mathcal{T}_{ll}$ is the trace of the subgrid-scale tensor $\mathcal{T}_{ij} = \bar{\rho}\tilde{u}_i\tilde{u}_j - \overline{\rho u_i u_j}$. The latter is related to the macro-pressure ϖ by the relation $\varpi = \bar{\rho}R\vartheta + (1/6)(3\gamma - 5)\mathcal{T}_{ll}$. In this relation, the last term is small even at high Mach, so that we neglect it and use the law of ideal gases between $\varpi, \bar{\rho}$ and ϑ . After some other approximations, one obtains a system equivalent to compressible Navier-Stokes equation for $\tilde{u}_i, \bar{\rho}, \varpi, \vartheta, \tilde{e}$, most of the molecular-diffusion coefficients being complemented by an eddy counterpart which is the same as in the incompressible case. We work with the filtered structure-function model. We first show an animation of quasi-longitudinal vortices travelling in a channel at low Mach. Then one studies a compressible round jet at Mach 0.7 and 1.4 forced upstream by a close to top-hat velocity to which a random perturbation is superposed. The Reynolds number is 36000. We show on the above figure a picture of the jet (left, Mach 0.7 ; right Mach 1.4) in the case of a white-noise forcing : the supersonic jet is much more focussed in space than the subsonic one, with an increase of the potential core. With other types of forcings, one can generate Reynolds blooming jet.

References

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