# INSTABILITY OF COROTATING VERTICAL VORTICES IN A STRATIFIED FLUID : WHY STRONGLY STRATIFIED TURBULENCE IS NOT SIMILAR TO 2D TURBULENCE

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*Summary* We present numerical, theoretical and experimental studies of a new instability of two corotating vertical vortices in a vertically stratified fluid. This instability induces the formation of thin horizontal layers with a thickness inversely proportional to the Brünt-Väisälä frequency. This three-dimensional instability is believed to make stratified turbulence depart from two-dimensional turbulence since it alters the pairing of vortices.

## INTRODUCTION

The atmosphere, the stratosphere and the ocean are characterized by a stable stratification that limits vertical motions of fluid. As a result, the velocity field is mainly horizontal and Riley et *al* [8] have shown that if both the horizontal and the vertical scales of the flow are large enough, the leading order dynamics are two dimensional. Lilly [4] has proposed that the kinetic energy spectra observed in the atmosphere at mesoscale are manifestation of the two dimensional dynamics with a transfer of energy from small (100 m) to large (100 km) scales. Recently, Lindborg [5] invalidated this interpretation using high order statistical moments and showing that the energy cascade was in the opposite direction : from large to small scales. Billant & Chomaz [2] and Lindborg [6] proposed that the dynamics were not two dimensional because the selected vertical scale at each step of the cascade was the local buoyancy length scale U/N (where U is the velocity of the vortices at the scale considered and N the Brünt-Väisälä frequency) which is small, invalidating the hypothesis of Riley [8]. Billant & Chomaz [1] also proposed that the vertical scale selection was due to an instability, named zigzag instability, able to decorrelate the flow on the vertical. They demonstrated this instability on the specific case of a counter-rotating vortex pair that is seldom encountered in turbulent flows. On the contrary, pairing of same sign vortices is believed to be the dominant process of two dimensional turbulence and to account for both the upscale energy transfer and the downscale enstrophy cascade. In the present paper, we extend the work of Billant & Chomaz and study the stability of a pair of corotating vortices in a stratified fluid. We show that the zigzag instability also affects the pairing of vortices and decorrelates the flow on the buoyancy length scale.

## NUMERICAL STABILITY ANALYSIS

A numerical stability analysis is performed using a pseudo-spectral solver of the Navier-Stokes equations linearized around a vertically uniform basic state made of two corotating Lamb-Oseen vortices (figure 1a). A Krylov method is implemented to extract the leading eigenmodes. For weak stratification, we found two unstable modes with distinct symetries that are due to the elliptic instability as described in the unstratified case by Le Dizès and Laporte [4]. An excellent agreement has been obtained with the latter study. For strong enough stratification, the elliptic modes are inhibited and a new mode takes over. As shown on the vertical vorticity field of the eigenmode (figure 1b), the perturbation corresponds to a symmetric displacement of the two vortices at an angle with respect to the x-axis. This mode is similar to the zigzag type since it translates the vortices without almost no deformation bringing them closer or further, alternatively along the vertical.

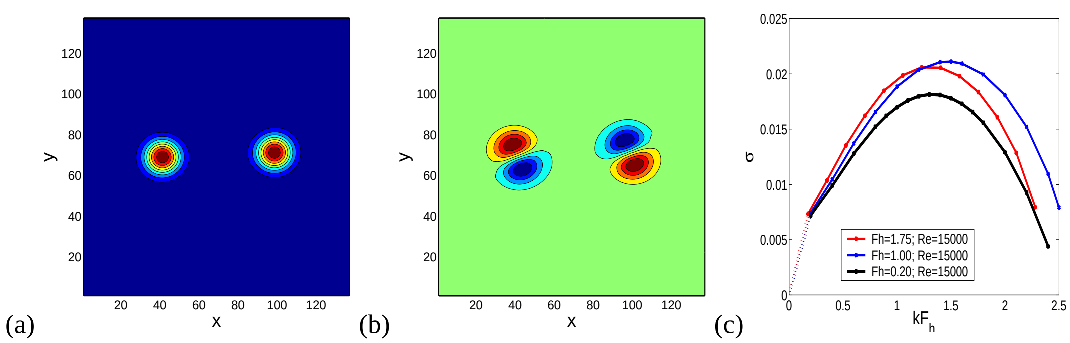


**Figure 1.** (*a*) Vertical vorticity of the basic state, (*b*) Vertical vorticity of the most unstable mode in a stratified fluid, (*c*) Growth rate against wave number times the horizontal Froude number for different Froude numbers.

Furthermore, the instability respects the scaling law found for zigzag instability, ie the most unstable wave number increases linearly with stratification (figure 1c).

## PHYSICAL EXPLANATION

This instability can be explained physically by the coupling between neutral waves on a single vortex in a stratified fluid and the strain field due to the companion vortex. If the corotating vortex pair is twisted along the vertical with respect to the rotation center, the motion induced by mass and potential vorticity conservation tends to bring the two vortices closer at the maximum of the twist. Thus, they rotate faster and the initial twist increases in an unstable way. A direct numerical simulation shows that the instability does not saturate due to non-linear effects so that vortex merging is speed up in some layers and inhibited in others resulting in a strong vertical decorrelation of the flow (figure 2a,b).

## EXPERIMENTAL OBSERVATION

The existence of this new instability has been confirmed experimentally with an apparatus consisting of two corotating flaps in a large stratified tank similar to the set up used by Meunier [7]. We can see on figure 2c that as in the DNS (figure 2a,b) the pairing is delayed in the top layer and accelerated in the bottom one.

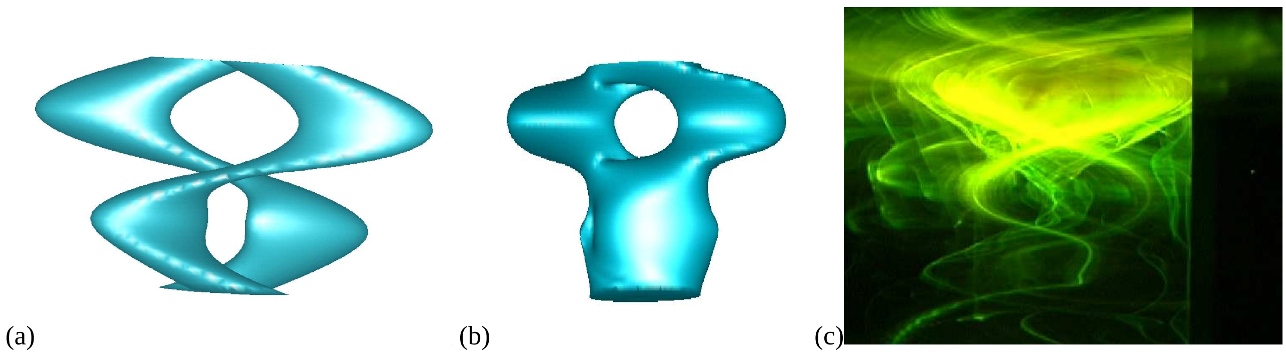


**Figure 2.** (*a*) DNS of the instability : 3D side view of a vertical vorticity contour. In the bottom layer, the vortices are getting closer whereas they are getting farther in the top layer. (*b*) The same as (*a*) later. The pairing has occurred in the bottom layer. (*c*) Experimental observation of the zig-zag instability. The green color is obtained by UV lighting and fluorescein initially painted on the edge of the flaps. When the flaps are closed, the fluorescein rolls up in the vortex cores. The eight shape visible on the picture is similar to the DNS result (figure 2a). It is the result of the zigzag instability that twists the vortex pair, promoting the pairing in the bottom layer and delaying it in the top layer.

## CONCLUSION

A new zigzag instability has been studied in the case of corotating vertical vortices in a stably stratified flow. The numerical stability analysis has shown that the instability consists in displacement of the vortices and selects a vertical wave length proportional to the buoyancy length ($L_B$=$U/N$). The coupling between the displacement mode of a vortex with the strain field generated by its companion is identified to be the physical mechanism of the instability. Experimental observation and DNS show that the instability does not saturate and leads to the vertical decorrelation of the flow on a scale proportional to $L_B$. As vortex merging plays a crucial role in two-dimensional turbulence, this new instability is believed to alter the energy transfer and therefore make three-dimensional stratified turbulence depart from two-dimensional turbulence.

## References

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