STRONG VORTEX INTERACTIONS IN QUASI-GEOSTROPHIC FLOWS

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Summary We examine the strong interaction two quasi-geostrophic vortices. The interaction depends on 6 parameters: the volume ratio, the potential vorticity ratio, the height-to-width aspect ratios, and the vertical and horizontal offsets. The parameter space is huge and highly efficient methods are needed to cope with the size of the problem. We primarily use a novel solution method, the Ellipsoidal Model (ELM), which models vortices as ellipsoids and filters higher-order deformations. It proves to be highly accurate and allows us to determine steadily rotating equilibria for both like-signed and opposite-signed vortices. We next determine the margin of linear stability of these states which we associate with the critical distance for strong vortex interactions. We complete the description of the interactions by illustrating the nonlinear evolution of delected unstable interactions here using the full dynamical equations (including non-ellipsoidal deformations).

INTRODUCTION

Vortical structures are frequently observed in the Earth's atmosphere and oceans. Vortices or coherent volumes of anomalous potential vorticity (hereinafter referred to as PV) play an important dynamical role in these environments since the dominant 'balanced' flow can be recovered from the knowledge of the PV distribution alone, cf Hoskins, McIntyre & Robertson (1985). In the adiabatic and inviscid fluid studied here, the PV is materially conserved and gives rise to a layerwise two-dimensional motion (there is negligible vertical advection due to the stable density stratification and the background rotation of the Earth). Coherent regions of PV are naturally associated with vortices. It is important to determine how nonlinear these structures interact since they are a key feature of atmospheric and oceanic dynamics. *Two-dimensional* vortex interactions have been extensively studied in the literature both for co-rotating and counter-rotating vortices, see Dritschel (1995) and references therein. However, in the context of *three-dimensional* quasi-geostrophic flows, which are more relevant to the atmosphere and oceans, little is known about strong vortex interactions despite previous research. The main reason for this lack of knowledge is arguably the size of the parameter space of the problem. Previous studies had to restrict attention to small parts of the full parameter space, normally considering situations in which numerous symmetries were imposed in the initial conditions, see *e.g.* von Hardenberg *et al.* (2000), Dritschel (2002) and Reinaud & Dritschel (2002). *We still do not know how vortices interact in general.*

To tackle this formidable problem, we first introduce a novel solution method: The Ellipsoidal Model (ELM) described in Dritschel, Reinaud & McKiver (2003) which gives the equations for the evolution of interacting uniform PV ellipsoids. The ELM is a finite Hamiltonian system given by

$$\frac{\mathrm{d}\boldsymbol{X}_i}{\mathrm{d}t} = -\frac{1}{\kappa_i}\mathcal{L}\frac{\partial H}{\partial \boldsymbol{X}_i}, \tag{1}$$

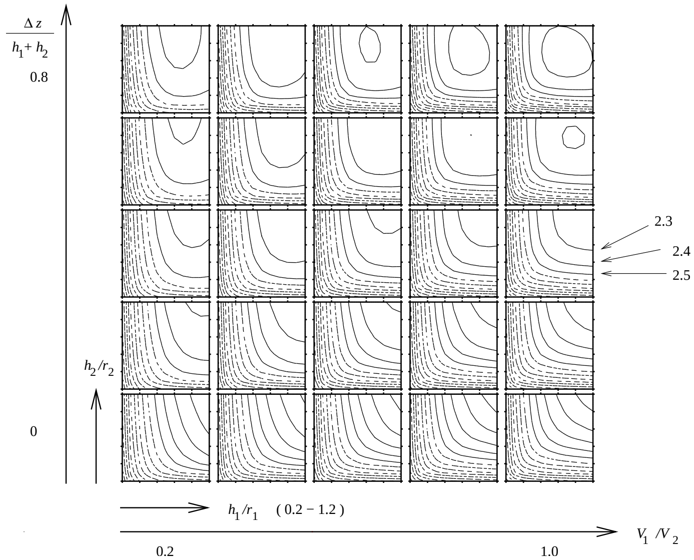
$$\frac{\mathrm{d}\mathcal{B}_i}{\mathrm{d}t} = \mathcal{S}_i\mathcal{B}_i + \mathcal{B}_i\mathcal{S}_i^T, \quad \text{and} \quad \mathcal{S}_i = -\frac{10}{\kappa_i}\mathcal{L}\frac{\partial H}{\partial \mathcal{B}_i}, \tag{2}$$

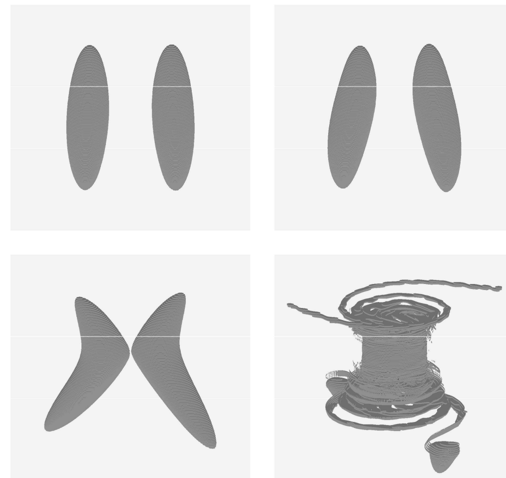
where $\boldsymbol{X}_i = (X_i, Y_i, Z_i)$ is the centroid of the i^{th} ellipsoid, and $\mathcal{B}_i$ denotes the 3×3 symmetric matrix in terms of which the boundary of the ellipsoid is expressed by $(\boldsymbol{x}-\boldsymbol{X}_i)^T\mathcal{B}_i^{-1}(\boldsymbol{x}-\boldsymbol{X}_i) = 1$. Also, κ_i is the 'strength', i.e. the volume integral of PV divided by 4π, and H is the Hamiltonian of the system, namely the total energy of the system divided by 4π. Finally, $\mathcal{S}_i$ is the 'flow matrix' depending on all ellipsoids. From these equations, we have derived a linearized system to compute equilibrium states by an iterative method. The states are then subjected to a linear stability analysis derived from the ELM equations. The low computational load required by the ELM enables us to study vortex interactions over wide ranges of the 6 parameters. Here, we primarily focus on like-signed (*equal-PV*) and opposite-signed (*opposite-PV*) interactions. We determine the margin of stability for vortices having aspect ratios consistent with what is actually observed in turbulence, namely $0.2 < h/r < 1.6$, see Reinaud, Dritschel & Koudella (2003). We perform the study considering 5 different values for both the vertical offset and the volume ratio.

Moreover we investigate the temporal evolution, *solving the full equations*, of characteristic interactions. We wish to determine the outcome of such vortex interactions and to study the way that both the energy and the enstrophy cascade during interactions and how this relates to what is observed in quasi-geostrophic turbulence.

RESULTS

We present here a sample of the results which have been obtained. The total volume of PV is set to $4\pi/3$. Contour plots of the critical distance between the vortex centroids are given in figure (1a) in a large parameter space. Results indicate for example that vortices moderately offset in the vertical (by about 1 mean radius) are likely to strongly interact for larger separation distances. Another important discovery is that prolate vortices are affected by a newly discovered instability: the *tilt* instability which has been overlooked by previous studies.





This instability appears first at distances for which vortices were thought to be stable in previous studies. It consists of the collapse of the vortices toward one another. Such an instability is illustrated in figure (1b) by a Contour Dynamics simulation of two vertically aligned vortices with $h/r = 4$.

As for the opposite-signed vortex interactions, a rather surprising result has been found. Only a few interactions are unstable according to the ELM. I.e., most of the cases investigated over a large parameter space are stable. For example, oblate vortices of equal volume and aligned in the vertical are stable even when they are touching. It is however not clear at the moment if such situations are genuinely stable or only stable to ellipsoidal perturbations. The equilibria might still be unstable to non-ellipsoidal perturbations – modes disregarded by the ELM. On the other hand, weak ellipsoidal instabilities may be observed for prolate vortices. Considering different examples of opposite-signed interactions none of them have been found to be particularly destructive. After a small amount of debris is cast off, the vortices return to a near equilibrium. An interesting result is found for vortices which are offset in the vertical by more than about $\frac{1}{2}(h_1+h_2)$. For these interactions, oblate vortices are likely to be unstable to ellipsoidal modes whereas prolate vortices are seen to be stable – the opposite situation to the one obtained for vortices aligned in the vertical or only slightly offset.

CONCLUSION

A combination of accurate and efficient numerical methods enable us to provide the first comprehensive study of vortex interactions in quasi-geostrophic flows. We use a promising new numerical model developed in Dritschel *et al.* (2003) that models vortices by ellipsoids of uniform potential vorticity. This model allows us to determine the margin of stability for equilibrium states that coincides with the critical distance for strong interactions. More than this, using the well-established CASL algorithm, we can study the outcome of these interactions and study how energy and enstrophy are redistributed spectrally as a result of the interactions.

References

[1] Hoskins, B. J., McIntyre, M. E. & Robertson, A. W. 1985 On the use and significance of isentropic potential-vorticity maps. *Q. J. Roy. Meteorol. Soc.* **111**, 877–946.

[2] Dritschel, D. G. 1995 A general theory for two-dimensional vortex interactions. *J. Fluid Mech.* **293**, 269–303.

[3] von Hardenberg, J., McWilliams, J. C., Provenzale, A., Shchpetkin, A. & Weiss, J. B. 2000 Vortex merging in quasi-geostrophic flows. *J. Fluid Mech.* **412**, 331–353.

[4] Dritschel, D. G. 2002 Vortex merger in rotating stratified flows. *J. Fluid Mech.* **455**, 83–101.

[5] Reinaud, J. N. & Dritschel, D. G. 2002 The merger of vertically offset quasi-geostrophic vortices. *J. Fluid Mech.* **469**, 287–315.

[6] Dritschel, D. G., Reinaud, J. N. & McKiver, W. J. 2003 The quasi-geostrophic ellipsoidal model. *J. Fluid Mech.* **in press**.

[7] Reinaud, J. N., Dritschel, D. G. & Koudella, C. R. 2003 The shape of the vortices in quasi-geostrophic turbulence. *J. Fluid Mech.* **474**, 175–192.