WAVE–VORTEX INTERACTIONS IN THE ATMOSPHERE AND OCEANS; WITH APPLICATIONS TO CLIMATE

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Summary Our aim is to address the following question: can we construct an accurate atmospheric climate model with a balanced model representing its fluid mechanics or dynamical core, and with dissipative as well as non-dissipative parameterization schemes for the gravity-wave activity? To do so, we therefore review the concepts of potential vorticity, balanced flow, linear waves and wave-vortex interactions.

INTRODUCTION

In the atmosphere, there is often a large separation in time scales between the slow large-scale motion of air and the rapid small-scale three-dimensional turbulence, the air motion around particles such as water droplets, and the kinetics of chemical trace gases. Knowledge of the chemical constitution of the atmosphere is required to understand the radiative damping and solar forcing of the atmosphere (Andrews *et al*., 1987). Also in the oceans, the time scale separation between the large-scale dynamics of water and the small-scale dynamics of turbulence and particle motions is vast. Numerical weather and climate prediction models of the atmosphere and oceans are therefore usually formulated as primitive equations for the motion air and water, respectively, for which the influence of (large quantities of) small-scale particles and chemicals is parameterized.

Even though we approximate the atmospheric and oceanic systems separately as single-phase systems, the small-scale quasi two-dimensional and three-dimensional wave and turbulent motion cannot be captured adequately by the numerical models. Consequently, also the feedback of the unresolved wave and turbulent motion on the large-scale dynamics requires parameterization. Only the large-scale dynamics is thus captured approximately in weather and climate prediction.

In addition, the dynamics of the atmosphere and oceans occurs on different time scales. Nevertheless, their large-scale dynamics share important characteristics and include similar asymptotic regimes in particular in terms of small Rossby- or Froude numbers. Hence, atmospheric and oceanic dynamics can be described to some extent in a unified way.

Of central importance in atmospheric and oceanic fluid dynamics is the (linear) dynamics of gravity-waves and vortical modes, the concept of potential vorticity, and the approximate or balanced description of the slow, large-scale dynamics in terms of the rotational or vortical motion. The advantage of these three items lies in their conceptual and mathematical clarity. The disadvantage is that the actual dynamics is more complicated, nonlinear and turbulent, and that the vortical motion is only approximately balanced. We introduce the above items for the simplified rotating Navier-Stokes equations by ignoring viscosity, invoking hydrostatic balance and using a layer-wise stratification.

LINEAR WAVES AND POTENTIAL VORTICITY

Consider the equations of motion for a one-and-half layer tropospheric-stratospheric model on the f-plane (Bokhove and Verkley, 2003)

$$\frac{\partial \sigma_\alpha}{\partial t} + \boldsymbol{\nabla}\boldsymbol{\cdot}(\sigma_\alpha\, \mathbf{v}_\alpha) = 0 \qquad \text{and} \qquad \frac{\partial \mathbf{v}_\alpha}{\partial t} + (\mathbf{v}_\alpha\boldsymbol{\cdot}\boldsymbol{\nabla})\mathbf{v}_\alpha + f\,\hat{\mathbf{z}}\times\mathbf{v}_\alpha = -\boldsymbol{\nabla} M_\alpha \tag{1}$$

with $\alpha = 2$ in the active lower tropospheric layer, horizontal coordinates x and y, horizontal gradient $\boldsymbol{\nabla}$, time t, $\hat{\mathbf{z}}$ the unit vector in the vertical, the pseudo-density $\sigma_2 = (p_2 - p_1)/g$ in the lower layer, the horizontal velocity $\mathbf{v}_2 = \mathbf{v}_2(x, y, t)$ in that layer, p_2 the pressure above the topography and p_1 the pressure at the stratosphere-troposphere interface, gravitational acceleration g, Coriolis parameter f, and Montgomery potential $M_2 = M_2(p_2)$. The lower tropospheric layer is asymptotically decoupled from the upper stratospheric layer since its thickness is much smaller than that of the deep overlying layer. This Montgomery potential M_2 is

$$M_2 = c_p\,\theta_2\,(p_2/p_r)^\kappa + g\,z_2, \tag{2}$$

where $\kappa = R/c_p$, the gas constant $R = c_p - c_v$, and $c_{p,v}$ is the specific heat at constant pressure and volume, respectively, p_r is a reference pressure, $z_2 = z_2(x, y)$ the vertical position of the topography, and θ_2 is the constant potential temperature related to the constant entropy in the lower layer. The model is closed by showing that $M_2 = M_2(\sigma_2)$, which follows from constraining the Montgomery potential in the decoupled stratospheric upper layer to be constant $M_1 = M_1(p_1, p_2) = cst$ (Bokhove and Verkley, 2003). After linearizing (1) around a resting basic state, in particular for constant f, the linear waves emerging consist of a pair of dispersive gravity waves with a frequency $\omega \geq f$, and a vortical mode with $\omega = 0$. The isentropic model (1) conserves energy and mass, and potential vorticity $q_2 = (f + \hat{\mathbf{z}}\cdot\boldsymbol{\nabla}\times\mathbf{v}_2)/\sigma_2$ is materially conserved such that $\partial_t q_2 + \mathbf{v}_2\cdot\boldsymbol{\nabla} q_2 = 0$. The one-and-half layer tropospheric-stratospheric model (1) can be extended to a model with $(N + 1/2)$ isentropic layers, also on the sphere.

BALANCED MODELS

When the stratification and the background rotation due to the Coriolis effect are relatively strong, the Froude and Rossby numbers are small, respectively, and balanced models can predict the large-scale vortical dynamics in an accurate and sometimes surprisingly accurate way (McIntyre and Norton, 2000). Within the phase space of the primitive equations of motion, *e.g.* (1), balanced vortical dynamics occur on a slow manifold devoid of acoustic and gravity waves.

In addition, we emphasize the nonlinearity in the dynamics in the inviscid limit by using Hamiltonian techniques. The preservation of conservation laws or even the Hamiltonian structure of balanced models in this limit is favored on heuristic grounds because of the assumed importance of conservation laws as constraints on the long-time (climatological) dynamics. Extending Vanneste and Bokhove (2002), Hamiltonian balanced models can be derived for equations (1) and its counterparts on the sphere (Buitendijk 2003). Since the Rossby and Froude numbers are not always small, and since the dynamics on the slow manifold is only approximate even for small Rossby and Froude numbers, the gravity-wave and vortical dynamics are interacting, often only weakly, but intermittently and locally also strongly.

WAVE–VORTEX INTERACTIONS

Although current weather and climate prediction models do resolve some of the (large-scale) gravity wave motion, the necessity to parameterize most of the gravity wave activity (also in the foreseeable future) suggests that we could instead use a global balanced model in which the gravity waves are fully parameterized or simplified. Olaguer (2002) built a global balanced climate model, including transport and chemistry, and tested it against other climate models. This comparison was reasonably good. Olaguer (2002) used the (stratified) vertical vorticity equation and the temperature equation as dynamical core using hydrostatic balance and global quasi-geostrophic balance.

In most current parameterizations (McFarlane, 1987; and such as used in Olaguer, 2002), gravity waves are for simplifying reasons only considered as forcing to the large-scale motion by dissipative means at locations where gravity-wave breaking occurs. These gravity waves are forced by orographic wave-mean flow interactions, and shear or convective instabilities.

The approximate nature of balanced vortical dynamics, or more precisely the lack of an exact slow (vortical) manifold, is exemplified by the generation of gravity waves by the vortical dynamics (Ford *et al*, 2000; Vanneste, 2004). In contrast to the orographic generation of gravity waves and the subsequent dissipative forcing of the large-scale flow, these gravity waves emerge from and leave an impact on the large-scale motion in a non-dissipative way. A key question is how we can parameterize these non-dissipative means of gravity-wave emission and absorption, and how important they are in forcing the mean, nearly balanced flow. The various mechanisms to generate and absorb gravity waves have received a lot of attention lately (Bühler, 2000; Bühler and McIntyre, 2003), and we will review some of these mechanisms with an eye on extending current parameterization schemes of (computationally unresolved) gravity waves.

PRELIMINARY CONCLUSIONS

We have started to address the following question: can we construct an accurate atmospheric climate model with a (Hamiltonian) balanced model as dynamical core and with dissipative as well as non-dissipative parameterization schemes for the gravity-wave activity? Olaguer (2002) uses an energy-preserving balanced model as dynamical core for climate predictions. Our preliminary conclusion is that this dynamical core can be replaced by a more accurate balanced model. Imposing an more accurate global balanced constraint, an Hamiltonian balanced model can be used as dynamical core extending existing results to the sphere (Buitendijk, 2003) and to stratified models. Alternatively, a stratified, mass conserving higher order balanced model could be used (*e.g.*, following McIntyre and Norton, 2000). One outstanding question is how the novel insights on the emission and absorption of gravity waves by the vortical motion can be incorporated efficiently in climate models. Another open question is how accurate and how much faster a climate model with a more advanced balanced model as dynamical core will be compared to existing climate models based on the hydrostatic primitive equations. The tests done by Olaguer (2002) with balanced vorticity and temperature equations as dynamical core and using hydrostatic and quasi-geostrophic balance appear, however, to be encouraging.

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