WAVES AND CURRENTS OVER A SEABED OF FINITE DEPTH

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Summary Laboratory experiments by earlier authors have shown that the near-surface velocity of an otherwise uniform current is reduced by following waves but is increased by opposing waves. By a two-dimensional boundary-layer analysis with depth-dependent eddy viscosity we show analytically that this is the consequence of second-order effects of wave-current interaction in two dimensions. Physical effects of waves on the current profile due to the moving free surface, wave attenuation and convective inertia are discussed. Comparisons with available experiments for smooth and rough seabeds are discussed. New predictions of the longitudinal variations along the current are made. The two dimensional theory is then extended to three dimensions, by examining the instability of the wave-affected current to periodic span-wise perturbations. In partiular we examine the initiation of longitudinal vortices similar to Langmuir circulation, but without wind. In addition to the internal vortex force first found by Craik & Leibovich [1], we shall show that nonlinearity induces a mean shear stress on the mean sea level, despite the absence of wind. This mean stress induces vertical vorticity downward to produce a new vortex force and can dominate the growth of longitudinal vortices. Effects of current strength and wave conditions on the unstable growth are studied by numerical examples.

INTRODUCTION

Fine sediments on the bottom of a shallow lake or sea can be resuspended by waves and transported by the current. Since these particles can be carriers of contaminants and nutrients, their distribution is crucial to the health of the water body. Quantitative understanding of the mutual influence between waves, current and wind is therefore of basic importance to the prediction of biological and/or chemical processes in shallow water. It is now also known in two dimensions that when waves and the current are in the same direction, there is also notable reduction of the mean current velocity near the water surface. Three-dimensional motion due to transverse instability of the two dimentional wave-current system can lead to longitudinal vortices similar to Langmuir cell and affects the transport of fine sediments.

We describe two- and three-dimensional theories of a moderately strong current modified by either current-following or current-opposing waves [5],[6]. Similar to earlier works [7] on wind profile above surface waves, we adopt an eddy-viscosity model in which the mixing length is measured from the moving water surface. Thus the eddy viscosity near the sea surface is modified by waves and depends on time. In consequence the mean wind velocity near the water surface is significantly affected. For two dimensions, the nonlinear interaction problem is solved analytically by a boundary-layer analysis . For three dimensions, the linearized instabilty of spanwise-periodic disturbances and the implicatons on longitudinal vortices are studied.

TWO DIMENSIONAL INTERACTIONS

We consider first the interactions between a steady current with velocity comparable to the orbital velocity of waves. Assuming that turbulence is dominated by the current, an eddy viscosity model is adopted which accounts for the moving free surface and different seabed conditions. In particular the eddy viscosity ν_e varies parabolically with depth, being zero at both the seabed and the moving free surface, and is the largest at mid core: $\nu_e = -\kappa u^*_{fc}(z-\zeta)\,(1+z/h)$. Inside the oscillatory boundary layer near the bed, the friction velocity must take a larger value [2]. After normalization and order estimates, a peturbation analysis is carried out. The equation governing the wave-perturbed current velocity in the core is found by including the effects of wave-induced Reynolds stress, and energy dissipation. Despite the relatively small velocity shear, the eddy-viscosity is large so that dissipation in the core is comparable to that in the bottom boundary layer. A key result at the second order is that, despite the absence of wind, the mean shear stress at the mean sea-level $z = 0$ is not zero, and is the combined effects of wave Reynolds stress, wave damping and the curvature of the eddy-viscosity. As a consequence the shear stress in the core is the sum of :(i) Surface distortion of the eddy viscosity, which is always negative and greatest on $z = 0$. (ii) Wave-induced change of friction velocity, whose sign depends on the directions of the waves and the current. (iii) Wave damping, which is always positive. (iv) Wave-induced Reynolds stress from the bottom boundary layer, which is always negative. (v) Curvature of the eddy viscosity, which is originated from the vortex force and is always positive. Sample comparisons with existing experiments on wave attenuation (Figure 1, Left) and on current velocity profiles (Figure 1, Right) are excellent.

LONGTIUDINAL VORTICES DUE TO INSTABILTY

Similar to Langmuir cells near the surface of a deep sea, longitudinal vortices are known to exist in shallow water. These cells are crucial to the mixing and transport of bottom sediments resuspended by waves. Instead of wind we shall examine an alternative mechanism where the pre-existing current is maintained either by tide or ambient pressure gradient. The water depth is taken to be comparable to the typical wave length, so that the current shear across the entire depth matters. We examine the instability of the two-dimensional wave-current system just described, to periodic disturbances in the

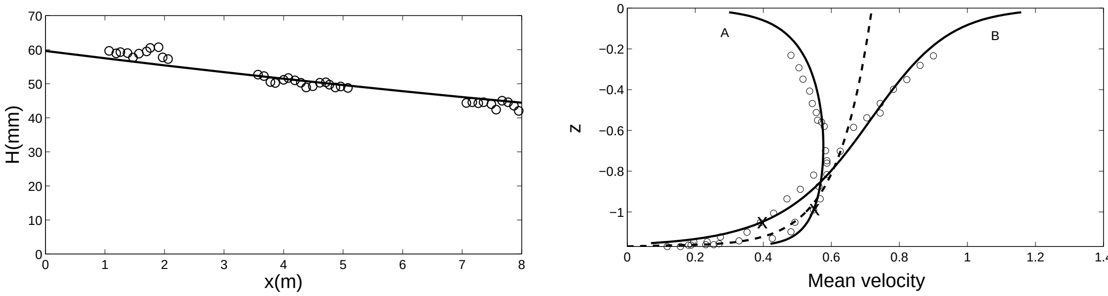

Figure 1. Left: Comparison of predicted wave height attenuation (solid line) with measurements for wave-opposing current [3]. Right: Comparisons of predicted velocity profiles (solid lines) with measurements [4]. The current is from left to right. A : waves are from left to right. B: waves are from right to left. Dashed line : theoretical profile of pure current.

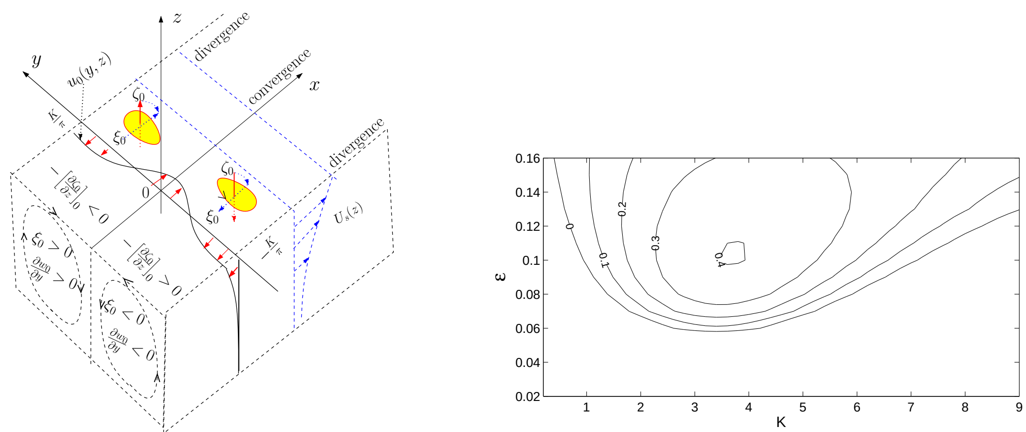

Figure 2. Left: Physical mechanism of instability. Right: Growth rate of an unstable mode as a function of wave steepness ϵ and lateral wavenumber K of periodic vortices.

span-wise direction. Using the same eddy-viscosity which varies with depth, the linearized equations governing along-crest perturbations are derived. Again the mean shear stress on the mean sea surface is found to be non-zero due to nonlinearity, despite the absence of wind. This surface stress feeds vertical vorticity downward and provides an additional source for longitudinal vorticity via the current shear, as sketched in Figure 2 (Left). If there is span-wise periodic divergence/convergence near the free surface toward lines parallel to the x axis, there must be upwelling and down-welling in a $y-$ period as sketched. Within a half $y-$period on the positive side, $\partial w_0/\partial y > 0$ in the core but vanishes on the free surface where $w_0 = 0$ everywhere. Hence its vertical gradient is negative on the free surface. Becasuse of the nonzero mean stress condition, there is a downward flux of positive vertical vorticity from the mean free surface $-\partial\zeta_0/\partial z < 0$. This influx is diffused downward to increase the vertical vorticity in the core. From the equation governing the longitudinal vorticity ξ, the increment of $\zeta_0 = -\partial u_0/\partial y$ induces further increase of the longitudinal vorticity ξ_0 through shear in the Stokes drift, hence leading to unstable growth. Within the half $y-$period on the negative side, all signs are reversed; a counter-rotating vortex in the $x-$ direction also grows. Thus the mean shear surface stress provides a source of vorticity from the upper boundary, in order for the mechanism of Craik & Leibovitch to take effect. Numerical results of an eigenvalue problem will show the eigenmodes corresponding to longitudinal vortices for typical field and laboratory scales. A sample plot showing the dependence of the growth rate on wave steepness ϵ and wavenumber K of peroidic vortices are shown in Figure 2 (Right). While instabilty is in principle possible in a wave-opposing current, it is in the wave-following current the unstable mode will be most effective in vertical mixing.

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