MODELLING OCEANOGRAPHIC COASTAL CURRENTS IN SMALL-SCALE AND LARGE-SCALE LABORATORY EXPERIMENTS

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Summary Laboratory experiments simulating gravity-driven oceanographic coastal surface currents are discussed. Results from two complementing studies on substantially different spatial scales and in different parameter regimes are compared. A geostrophic model is developed in terms of a set of non-dimensional parameters obtained from dimensional analysis. Very good agreement with experiments is found.

INTRODUCTION

When estuarine river water discharges into the coastal zone a gravity-driven surface flow is established. The flow develops as a consequence of the density difference between the discharged, buoyant fresh water and the denser, salty ocean water. The flow can be affected by the Coriolis force arising from the rotation of the earth. This confines it to the coastal zone where it forms a current flowing along the coast [1-3]. To date there exists no complete model which can predict the current width, its height and its propagation speed on the basis of a simple geostrophic approximation. The purpose of this study was to develop such a model and test it against comprehensive experimental laboratory data.

THEORETICAL ANALYSIS OF THE FLOW

Dimensional analysis

Five quantities are involved; dimensional analysis yields three non-dimensional parameters to summarise the data:

$$I = q_0^{1/5}\, \Omega\, g'^{-3/5}, \qquad T = g'^{3/5}\, t\, q_0^{-1/5}, \qquad \Pi(X) = X\, g'^{1/5}\, q_0^{-2/5} \tag{1a-c}$$

(q_0 : volumetric discharge rate at source, Ω : rotation rate, g' : reduced gravitational acceleration $g' = (\rho_2 - \rho_1)\, g/\rho_1$ with ρ_1 and ρ_2 representing densities of fresh and ocean water respectively and g is the gravitational acceleration, t : time). Eq. (1a) summarises the independent experimental parameters and characterises experiments in parameter space. Eq. (1b) represents a non-dimensional time. Eq. (1c) is a non-dimensional length; where X represents, alternatively, the current length $l(t)$, its width w_0 or its height h_0 (see Fig. 1). We write $\Pi(l) = L$, $\Pi(w_0) = W_0$ and $\Pi(h_0) = H_0$.

Theoretical model for the current

Our model assumes that the flow velocities normal to the wall and in the vertical direction (see Fig. 1) are negligible in comparison to the along-wall flow velocity. We allow for motion along the x-axis but neglect all variations $\partial/\partial x$.

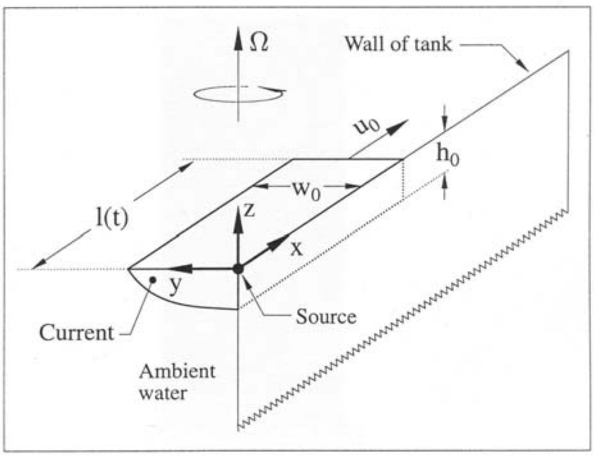

Fig. 1 : Sketch to illustrate nomenclature for model.

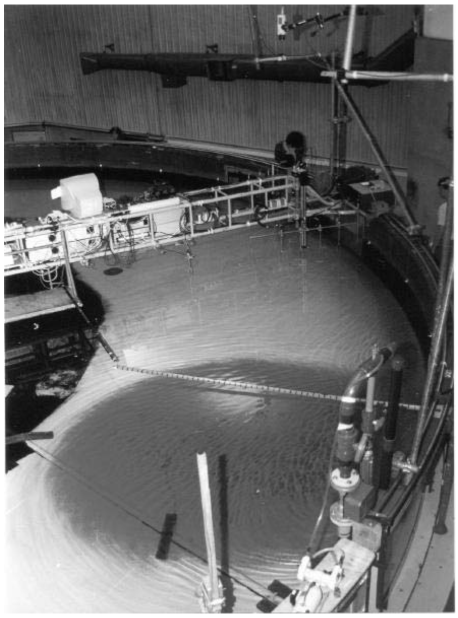

Fig. 2 : A current in the large-scale facility.

We use the geostrophic approximation, assume that the potential vorticity is zero and conserved and we employ mass conservation. The theoretical analysis then reveals that the most appropriate time scale is $T_0 = I^{1/4}\, T$ and that

$$H_0 = 2 I^{1/2}, \qquad W_0 = I^{-3/4}, \qquad L = \frac{3}{4} T_0, \qquad U_0 = \frac{L}{T_0} = \frac{3}{4}. \tag{2a-d}$$

Equations (2a,b) yield $h_0 / w_0 = 2\, I^{5/4}$. One can define a Rossby number Ro and a Froude number Fr and finds

$$Ro = \frac{u_0}{\Omega\, w_0} = \frac{3}{4}, \qquad Fr = \frac{u_0}{\sqrt{g'\, h_0}} = \frac{3}{2^{5/2}} = 0.5303 \tag{3a,b}$$

The results expressed by Eqs. (2a-d) and (3a,b) have not appeared in the literature previously. The expressions can be tested against experimental data.

THE EXPERIMENTS

Two complementing experimental studies were carried out. One study was conducted in a small rotating tank with diameter 1 m. The second was on much larger spatial scale; it employed the world's largest rotating turntable at the Coriolis Facility (Grenoble) with its 13-metre diameter tank [4]. For the experiments the tank was filled with dense salt water. This was brought into solid-body rotation and represented the ocean. Fresh water, simulating river discharges, was released continuously from a source mounted at the wall of the tank. The source was adjusted to be level with the surface of the salt water. The fresh water was dyed with food coloring to enable distinguishing the current from the ambient clear water. Defining a Reynolds number $\mathrm{Re} = w_0\, u_0/\nu$, ($\nu$: kinematic viscosity) conditions were such that $116 \le \mathrm{Re} \le 5223$ and $5007 \le \mathrm{Re} \le 185542$ for the small- and large-scale experiments respectively.

EXPERIMENTAL RESULTS

Qualitative observations

Figure 2 shows part of a large-scale current. The current fluid is dyed red. The source from which the fluid ejects is located at the wall of the tank near the lower right-hand corner of the photo. The currents in the small-scale facility look qualitatively very similar. A total of 34 large-scale and 66 small-scale experiments were conducted and analyzed. Depending on the experimental conditions currents can be stable or unstable. Figure 2 shows a stable current. Unstable currents are characterized by the development of baroclinic instabilities establishing eddies on the currents.

Quantitative results

Figure 3 serves as one example illustrating the accuracy with which our geostrophic model describes the current dynamics. The figure summarises the data for the development of the non-dimensional current length L as a function of the non-dimensional time T_0 for the 34 large-scale experiments. The solid line superposed on the data represents the theoretical prediction of Eq. (2c). It can be seen that the model describes the experimental data very well. The currents are initially slightly faster than predicted because their height decreases along the current from the source to the current head. The corresponding figure for the 66 small-scale experiments looks essentially identical to Fig. 3. All figures for the current height H_0 and the current width W_0 as a function of the parameter I display a similarly good agreement with theory when experimental conditions are geostrophic. The data show how agreement becomes less favourable as ageostrophic experimental conditions are approached.

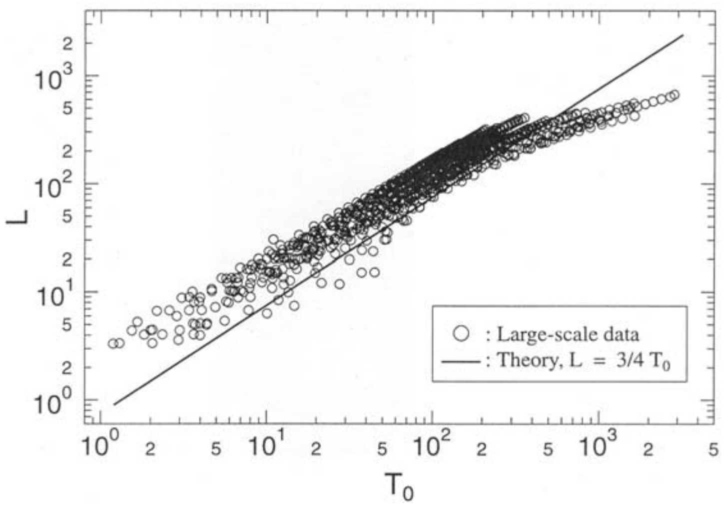

Fig. 3: Summary of experimental data for the current length L as a function of time T_0 for the 34 large-scale experiments.

SUMMARY AND CONCLUSION

A geostrophic model describing gravity-driven buoyant surface currents in a rotating system was developed. Comparisons with data from small-scale and large-scale experiments show good agreement in the geostrophic parameter regime. Discrepancies between model and experiment are associated with the surface Ekman layer which is not governed by geostrophy. The agreement between measured and predicted current height scales with an Ekman number $Ek(h_0)$ defined on the basis of the current height h_0. Agreement between model and experiments deteriorates with increasing Ekman number as $Ek^{0.12}$. The current width appears to scale with a the ratio of $Ek(h_0)$ and the Reynolds number $\mathrm{Re}(w_0, u_0)$ defined on the basis of the current width w_0 and the current speed u_0.

References

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