CAPILLARY PRESSURE OF A LIQUID BETWEEN UNIFORM SPHERES ARRANGED IN A SQUARE-PACKED LAYER

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Summary The capillary pressure in the pores defined by equidimensional close-packed spheres is analyzed numerically. In the absence of gravity the menisci shapes are constructed using Surface Evolver code. This permits calculation the free surface mean curvature and hence the capillary pressure. The dependences of capillary pressure on the liquid volume constructed here for a set of contact angles allow one to determine the evolution of basic capillary characteristics under quasi-static infiltration and drainage. The maximum pressure difference between liquid and gas required for a meniscus passing through a pore is calculated and compared with that for hexagonal packing and with an approximate solution given by Mason and Morrow [1]. The lower and upper critical liquid volumes that determine the stability limits for the equilibrium capillary liquid in contact with square packed array of spheres are tabulated for a set of contact angles.

EXTENDED SUMMARY

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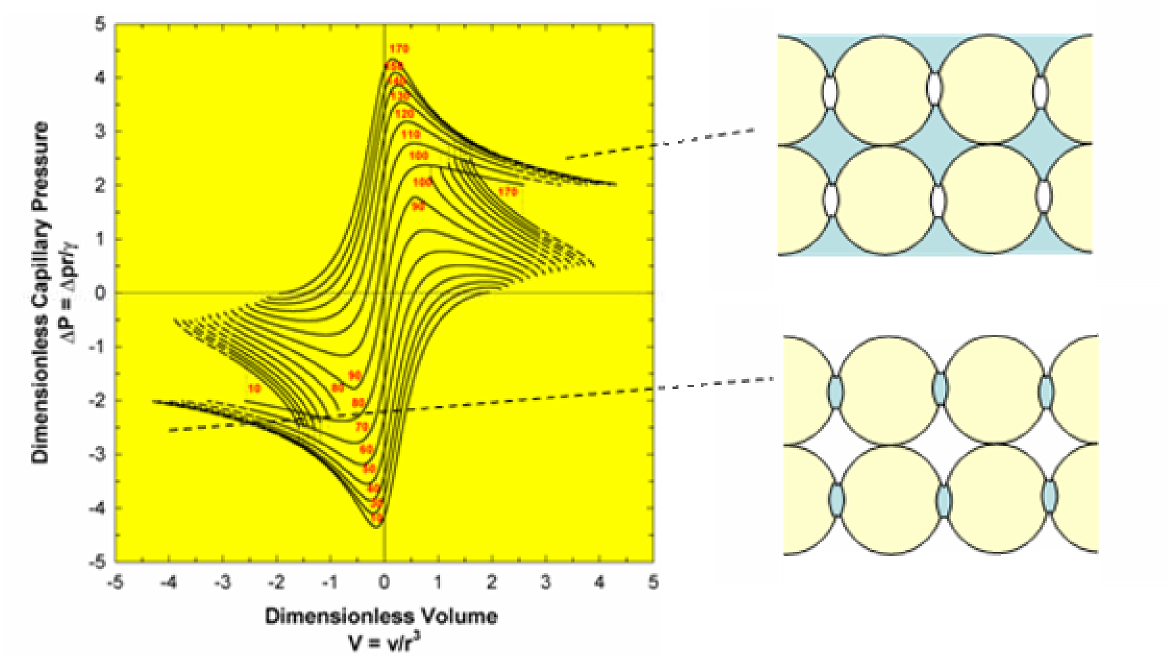

Fig. 1. Dependence of dimensionless capillary pressure, ΔP, on dimensionless liquid volume, V, (liquid on bottom) for pores formed by square close-packed spheres Numbers on curves denote the value of the contact angle θ (in degrees). For $\theta \neq 90°$, the dependence consists of two branches. The vertical dotted lines represent transitions between two branches of capillary surfaces. The segments shown by dash lines are valid only for a single-layer packing.

Our calculations show that the limiting cases of hexagonal and square packing diverged considerably. The maximum capillary pressures for $90° \le \theta \le 180°$ (the minimum capillary pressures for $0 \le \theta \le 90°$) differ by $1.75 \div 2.5$ times. There are also qualitative distinctions. In the case of hexagonal packing and $90° < \theta \le 180°$, the existence of two branches of equilibrium configurations has been illustrated in [2] only for $\theta = 100°$ and $110°$, and there is no evidence that the same holds for $120° \le \theta \le 180°$. For the case of $\theta = 110°$ that was analyzed in [2], the value of the dimensionless volume V at the extreme right point of the long branch (LB) is less than that at extreme right point of the short branch (SB), i.e. $V_p^* < V_r^*$.

For square packing, two branches exist for any $90° < \theta \le 180°$. Here $V_p^* < V_r^*$ if $90° < \theta \le 109°$, and $V_p^* > V_r^*$ if $110° \le \theta \le 180°$. This causes different ways of infiltration and drainage in a wide interval of θ. Under infiltration, a layer of a square packed arrangement becomes completely coated by liquid as a result of loss of meniscus stability if $90° < \theta \le 109°$, and as a result of a pendular gas ring formation if $110° \le \theta \le 180°$. Similar conclusions are valid for detachment of liquid from the spheres during drainage: loss of the capillary surface stability for the case of square packing and $71° \le \theta < 90°$, and formation of a pendular liquid ring in the case of $0 \le \theta \le 70°$.

In contrast to analysis of hexagonal packing [2], for square packing the stability boundaries for an equilibrium capillary liquid in contact with spherical array have been determined here with a reasonable accuracy and will be presented as tabulated data. It is notable that according to numerical results the modulus of mean curvature for a free surface with self-tangency is practically independent of θ.

For both hexagonal and square packing there is a hysteresis behavior in quasi-static infiltration and drainage processes. The hysteresis is due to irreversible transitions between configurations that belong to different branches, and between a horizontal free surface and a capillary surface in contact with spheres.

All descriptions are valid for mono-layer packing. It is significant that for any contact angle the menisci corresponding to maximum and minimum capillary pressures never cross spheres from the next layers of three-dimensional packing. We have verified this for both square and hexagonal packing. This suggests that for multilayer packing the extreme capillary pressures are the same as for mono-layer packing. Thus, our results allow for an estimation of the lower limit of penetration pressure for mixed types of packing.

References

[1] G. Mason, N. R. Morrow, J. Colloid Interface Sci. 168 (1994) 130-141.

[2] J. L. Hilden, K. P. Trumble, J. Colloid Interface Sci. 267 (2003) 463-474.