ONSET OF OSCILLATIONS IN HIGH-PRANDTL THERMOCAPILLARY LIQUID BRIDGES: LINEAR-STABILITY ANALYSIS VS. EXPERIMENT

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Summary The float-zone crystal-growth process is studied in the framework of the half-zone model. The linear-stability analysis is used to compare the onset of oscillations in liquid bridge thermocapillary convection upon liquid bridge volume with experimental data for high-Prandtl fluid. Well-known structure of neutral stability curve for high-Prandtl fluid, consisting of two branches separated by an "overstability" gap, has been clearly reproduced. Moreover, for low-volume branch there is a good quantitative correspondence between experimental data and linear stability analysis. Travelling hydrothermal waves with unit azimuthal wave number correspond to critical perturbations for both branches. An influence of the temperature-dependent viscosity, the heat loss through the free surface and the gravity level on the stability limits is determined.

Introduction

Numerous experiments on transparent high-Prandtl liquids show essentially non-monotonic behaviour of stability limit versus liquid-bridge relative volume: the stability diagram consists of two branches with an “overstability” gap. In different experiments the “overstability” gap corresponds to relative volume V $0.8 \leq V \leq 1.0$. The studies of high Pr flows suffer from practically the lack of direct comparisons of experimental data, linear-stability analysis and direct numerical simulation results. The reasons are severe numerical difficulties, preventing from the reliable modelling, and too many factors, affecting convection in real liquid bridge experiments. The lack of cross-checkings could excite reasonable doubts in reliability of the models applied and the results performed by linear stability analysis and direct numerical simulation. The goal of this paper is to apply linear-stability analysis to determine the stability limits for liquid bridge with highly deformed free surface versus liquid- bridge volume and to compare them with experimental data [1].

Formulation of the problem

The geometry considered consists of an axisymmetric liquid bridge, which is suspended between two horizontal co-axial rigid disks of equal radius r_0 in a distance d (Fig. 1). The lower and upper disks are kept at constant temperatures T_0-$\Delta T/2$ and T_0+$\Delta T/2$, respectively, where T_0 is the mean (reference) temperature. The lateral side of the liquid $r=h(z)$ is a free surface and it's shape is governed by the mutual action of the gravity and the surface tension. The governing equations are the Boussinesq equations for a Newtonian fluid with viscosity ν and surface tension σ assumed to be a linear function of temperature. At the rigid disks the boundary conditions are no-slip and no-penetration and the temperature is fixed. For the free surface the mixed thermal boundary condition is assumed and the velocity-boundary conditions are derived from the free-surface stress balance. The kinematic boundary condition at the free surface interface requires the normal velocity to be zero. Appropriate symmetry boundary conditions at r=0 complete the problem. Non-dimensional geometrical parameters are the aspect ratio Γ and the relative volume V. Other non-dimensional governing parameters are the Prandtl Pr, the Reynolds Re, the viscosity group R_v, the Bond Bo, the Grashof Gr, the Biot Bi numbers and the non-dimensional external temperature. The details of the problem statement, and methods of the linear stability problem solution could be found in [2].

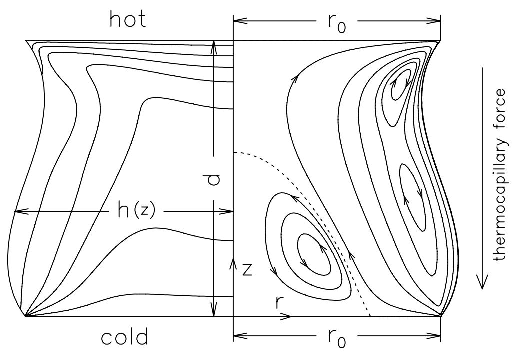

Fig. 1. The liquid bridge, the coordinate system (r,z), the fluid flow structure: the isotherms (left side)

Results and discussion

Detailed calculations have been carried out for the liquid-bridge experimental stability study [1]. The liquid-bridge had the fixed aspect ratio Γ=1.5, where as the relative volume V was varied from 0.5 to 1.4 and the temperature difference was varied up to 70 ºC. The 5 cS silicone oil used in experiment has a linear temperature dependence of the surface tension and an exponential temperature dependence of the kinematic viscosity. For numerics, the exponential behaviour is approximated by a linear dependence. The Fig. 2 shows the comparison of the experiment and linear stability analysis for temperature-dependent viscosity and constant viscosity cases for zero Biot number. Following the experimental data [1], the linear stability analysis reproduces two branches structure of the stability diagram with the "overstability" gap, in contrary to the energy-stability analysis has been additionally employed in [1]. Both temperature-dependent viscosity limits and constant viscosity limits differ insignificantly except the range of relative volumes V $0.7 \le V \le 1$, where there is the most slope of neutral curve. For the low-volume branch there is a good quantitative correspondence between the experiment and the numerical analysis for the temperature dependent viscosity case. However, for the high-volume branch there is only a good qualitative correspondence between experiment and numerical analysis. Travelling hydrothermal waves with unit azimuthal wave number correspond to critical perturbations for both branches. To improve the high-volume branch, the non-zero heat flux through the free surface is proposed. The Fig. 3 shows linear stability results for Biot number Bi=0.2 and external temperature equals the temperature of the cold bottom disk. The linear-stability results fit the experiment better now, however for relative volumes V $0.85 \le V \le 1.05$ the flow becomes to be much more stable. Note, that the heat flux through the free surface decreases the critical Marangoni number for high-volume branch. The reason is that a very flat temperature distribution on the free surface increases the slope due to the heat loss. This leads to flow enhancement and a loss of stability. Calculations for zero gravity level show small difference in the critical temperature difference less then 4 °C because of enough small Bond and Grashof numbers.

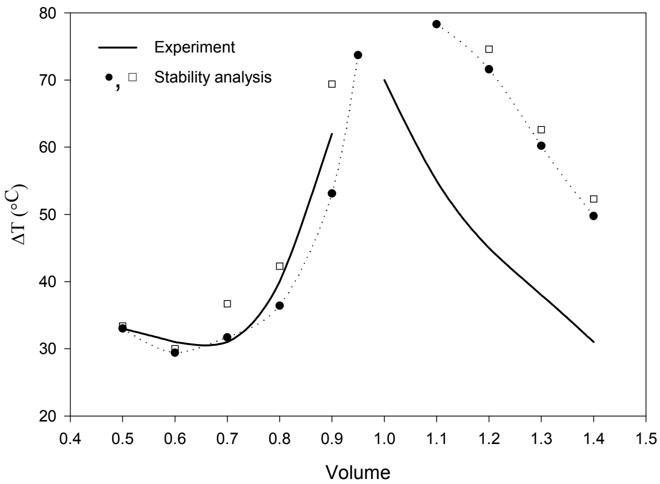

Fig. 2. Stability diagram: the critical temperature difference versus the liquid bridge relative volume. The solid lines correspond to the experiment [1], the dotted lines and filled circles correspond to the linear stability analysis to the temperature-dependent viscosity case, hollow squares - to the constant viscosity case for Bi=0.

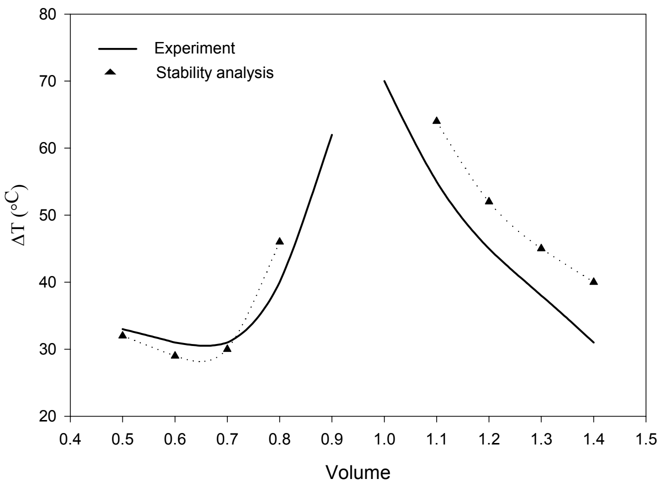

Fig. 3. Stability diagram: the critical temperature difference versus the liquid bridge relative volume. The solid lines correspond to the experiment [1], the dotted lines and filled triangles correspond to the linear stability analysis to the temperature-dependent viscosity case and Biot number Bi=0.2.

References

[1] Sumner L.B.S., Neitzel G.P., Fontaine J.-P., et al. *Phys. Fluids* 13 (2001) 107.

[2] Ermakov M.K., Ermakova M.S. *J. Crystal Growth* 266 (2004) 160.