RATCHETING-INDUCED WRINKLING OF AN ELASTIC FILM ON A METAL LAYER UNDER CYCLIC TEMPERATURES

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Summary This paper develops a theoretical model for wrinkling of an elastic film on an elastic-plastic metal layer caused by cyclic temperatures. Both analytical and numerical solutions will be presented. The implications of the results for structural evolution and failure mechanisms in integrated electronic devices and thermal barrier coating systems will be discussed.

INTRODUCTION

Under in-plane compression, a freestanding membrane tends to buckle into an equilibrium state. For a thin film bonded to a substrate, the buckling is constrained. If the substrate is elastic and relatively compliant, the film may still buckle into an equilibrium state. The wavelength of the equilibrium state, however, depends on the compressive stress in the film and the elastic compliance of the substrate, which is shorter than buckling wavelength of a freestanding film and often called wrinkling. If the substrate creeps, wrinkling becomes a kinetic process, as the amplitude grows over time [1-3]. At high temperatures, interfacial diffusion may also facilitate the kinetic process of wrinkling. This paper presents a theoretical study on another mechanism of wrinkling, which is induced by ratcheting plastic deformation under cyclic temperatures.
The deformation in the same direction caused by a cyclic load (e.g., thermal cycling) is known as ratcheting, which has been observed in many engineering structures. Recently, the concept of ratcheting has been used in modeling metal film crawling as well as induced cracking in integrated electronic devices [4-6]. An analogy between ratcheting and creep has been developed to understand the kinetics of ratcheting-induced crack initiation and growth [7]. The same analogy suggests that, under cyclic temperatures, a compressively strained elastic film can form wrinkles on a metal layer. Such ratcheting-induced wrinkling has been observed in thermal barrier coating systems.

THE MODEL

Figure 1 shows the model structure. The elastic film is modeled by the nonlinear von Karman plate theory with both normal and shear tractions at the film/metal interface. The deformation in the metal layer is a superposition of two parts. The thermal expansion mismatch between the metal and the substrate induces a biaxial in-plane stress, which is assumed to be large enough to cause the metal to deform plastically during each cycle. The compressively strained film tends to buckle away from the flat state, which induces normal and shear tractions along the metal/film interface. The interface tractions do not change directions within each cycle and biased the plastic flow at yield, leading to ratcheting deformation of the metal and growth of the wrinkle. Assuming an elastic-perfectly plastic behavior of the metal, we obtain the net displacements of the film per cycle

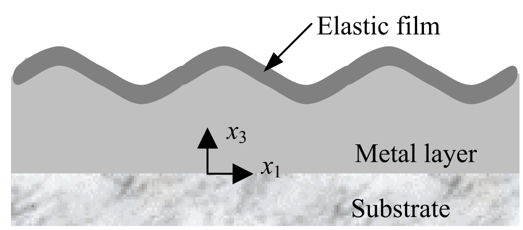

Figure 1: Model structure

$$\frac{\partial w}{\partial N} = \frac{1}{\eta_R}\frac{\partial}{\partial x_\alpha}\left[\frac{H^3}{3}\frac{\partial p}{\partial x_\alpha} - \frac{H^2}{2}\tau_\alpha\right], \qquad \frac{\partial u_\alpha}{\partial N} = \frac{1}{\eta_R}\left(\tau_\alpha H - \frac{H^2}{2}\frac{\partial p}{\partial x_\alpha}\right), \tag{1}$$

where w is the out-of-plane displacement, u_α the in-plane displacement ($\alpha = 1,2$), N the number of temperature cycles, H the thickness of the metal layer, p and τ_α the normal and shear tractions at the interface, respectively, and η_R is called ratcheting viscosity [5] as defined below:

$$\eta_R = \frac{E_m}{12(1-\nu_m)}\left[\frac{E_m(\alpha_m - \alpha_s)(T_H - T_L)}{(1-\nu_m)Y_m} - 2\right]^{-1}, \tag{2}$$

where α_m and α_s are the thermal expansion coefficients of the metal layer and the substrate, respectively, T_H and T_L define the temperature range of cycles, E_m, ν_m, and Y_m are the modulus, Poisson ratio, and uniaxial yield stress of the metal. The equilibrium of the elastic film relates the tractions at the interface to the current displacements by the von Karman plate theory. Together we have a set of partial differential equations that evolve the displacement field as the temperature cycles.

The equations in (1) are analogous to those obtained for an elastic film on a viscous layer [2], with the number of cycles N replacing time t and the ratcheting viscosity η_R replacing creep viscosity. The above equations also reduce to those obtained for metal film crawling in [5], where only a constant and uniform shear traction was applied at the surface of a metal layer. The present model suggests that, while a uniform shear traction at the metal surface causes metal to crawl, a non-uniform shear traction and/or pressure cause the metal to crawl and wrinkle simultaneously; on the other hand, a uniform pressure at the surface by itself does not cause any ratcheting deformation in the metal.

RESULTS AND DISCUSSIONS

Assuming a sinusoidal wrinkle of small amplitude, a linear perturbation analysis predicts that the amplitude grows as $A(N) = A_0[1+s(\lambda)]^N$, where A_0 is the initial amplitude, and the growth rate s depends on the wrinkle wavelength λ. There exists a critical wavelength λ_c, below which the wrinkle decays ($s < 0$) and above which the wrinkle grows ($s > 0$). The value of s reaches a maximum at λ_m, corresponding to the fastest growing mode. Starting from a random perturbation, the fastest growing mode will dominate the initial growth.

For any wavelength $\lambda > \lambda_c$, there exists an equilibrium wrinkling state, in which the tractions at the film/metal interface vanish. Consequently, the metal undergoes cyclic plastic deformation without ratcheting and wrinkle stops growing. The equilibrium amplitude of the fastest growing mode is $A_{eq} = h/\sqrt{6}$, where h is the film thickness.

A finite difference method was employed to numerically simulate the evolution of wrinkles under the plane strain condition. Figure 2 shows the growth of the amplitude of a sinusoidal wrinkle as the temperature cycles. The growth follows the prediction of the linear perturbation analysis for the first 500 cycles, and then saturates to approach the equilibrium amplitude. Next we start from a perturbation described by a Gauss function and assume an elastic constraint layer above the film (e.g., in a thermal barrier coating system). Depending on the stiffness of the elastic constraint, the wrinkle grows differently. Figure 3 shows two limiting cases: without any constraint, the wrinkle grows to a sinusoidal shape corresponding to the fastest growing mode; with a very stiff constraint layer, the upward displacement is completely suppressed, and the wrinkle grows downwards into a deeper and narrower shape.

Ratcheting-induced wrinkling may cause failure in integrated electronic devices and thermal barrier coating systems. Bending of the film induces tensile stress that may cause the film to crack. The normal traction at the film/metal interface oscillates between tension and compression, and a tensile traction may cause debonding of the film.

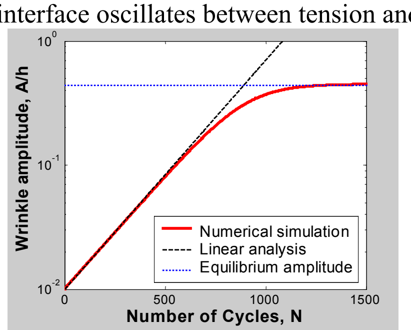

Fig 2: Growth of a sinusoidal wrinkle.

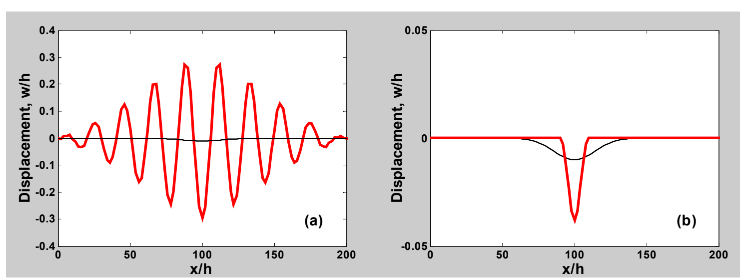

Fig 3: Wrinkling with (a) no constraint and (b) a very stiff constraint.

ACKNOWLEDGMENTS

The authors thank partial support from Microelectronics Research Center of The University of Texas at Austin. Discussions with Z. Suo of Harvard University were both encouraging and helpful.

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