HYDRODYNAMIC INTERACTION BETWEEN TWO BIOARTIFICAL CAPSULES IN SHEAR FLOW

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Summary The hydrodynamic interaction between two liquid filled identical spherical capsules is studied in simple shear flow. The boundary element method is used with bi-cubic B-splines as basis functions to map the capsule surfaces on two structured mesh. This guarantees continuity of second order geometrical properties with respect to the position of the Lagrangian particles used for tracking the location of the interface at each time step. When crossing, the capsules undergo large deformation that may damage the interface and their trajectories are irreversibly shifted towards larger cross streamline separation.

PROBLEM PRESENTATION

Liquid filled bioartificial capsules enclosed by a thin hyper-elastic membrane are used in many biomedical or industrial applications where encapsulation of living cells or of active agents in a protecting membrane is necessary. The design of such capsules has to take into account the mechanical stress to which the particle will be subjected in order to control burst. There have been a number of numerical and experimental studies of the motion and deformation of a *single* capsule freely suspended in shear flow. However, such particles are often used in suspension and it is then of interest to consider the interaction between several capsules. This is a difficult multi particle problem that has not been studied up to now to our knowledge. In this paper, we present the first numerical study of the hydrodynamic interaction between two identical capsules freely suspended in a simple shear flow.

Two identical initially spherical capsules C_1 and C_2, with radius a, are filled with a Newtonian fluid of viscosity μ and enclosed by a very thin hyperelastic membrane, devoid of bending resistance, and characterised by surface shear and dilation modulus G_s and K. The two capsules are suspended in another Newtonian liquid with same viscosity μ and subjected to a simple shear flow $\mathbf{u}^{\infty}=kx_3\mathbf{e_1}$ with shear rate k in the x_1, x_3 plane. Buoyancy effects are ignored and the fluid motion is governed by the Stokes equations. The centres of mass G_1 and G_2 of the capsules are initially located in the shear plane x_1, x_3 and remain in this plane of symmetry during motion. Since the capsules are identical, their deformed shapes are symmetric with respect to the middle of the segment G_1G_2. The relative position of C_1 and C_2 is given by the difference between the centres of mass position: $\Delta x_i=x_i(G_1)-x_i(G_2)$, with initial value denoted d_i.

The fluid velocity at any point $\mathbf{x_0}$ is given by the boundary integral on the surfaces C_1 and C_2 of the two capsules

$$\mathbf{u}(\mathbf{x}_0)=\mathbf{u}^{\infty}(\mathbf{x}_0)-\frac{1}{8\pi\mu}\int_{C_1\cup C_2}\mathbf{J}(\mathbf{x},\mathbf{x}_0).\Delta\mathbf{f}(\mathbf{x})\,dS(\mathbf{x}) \qquad (1)$$

where $\mathbf{J}$ is the free space Green's function [1]. The jump $\Delta\mathbf{f}$ in viscous traction across the interface is the elastic load on the interface and is thus related to the elastic tensions in the membrane through the shell equilibrium equations [1, 2].

Two membrane constitutive equations are considered [2]: the neo-Hookean (NH) law that corresponds to a very thin sheet of a volume incompressible elastomer or the Skalak et al. [3] two-dimensional law (SK). We consider the case where the two laws are identical for small deformation but lead to different behaviour for large deformation. Since the capsule interface has no bending stiffness, it cannot sustain any compressive stress and will buckle as soon as one of the two principal tensions becomes negative. A simple stability criterion is thus that the two principal tensions should both be positive. Dimensional analysis of the problem shows that an important parameter is the *capillary number* $\varepsilon=\mu k a/G_s$ which measures the ratio between the viscous forces exerted by the fluids and the elastic resistance of the membrane.

NUMERICAL PROCEDURE

The numerical method is based on Lagrangian tracking of the position of the interface material points: starting with the two undeformed capsules at some initial position, we follow their motion and deformation in response to a sudden start of the flow. Following the method developed for an isolated capsule [4], we use a structured mesh for each particle and interpolate the surface with bi-cubic B-spline functions. This guarantees continuity of second derivatives and curvature and allows the direct determination of the elastic load $\Delta\mathbf{f}$ on the interface. The integral in equation (1) is calculated by means of regular Gaussian quadrature. Once the velocity is calculated, the nodes on the surface are convected to their new position over time interval Δt by means of a fourth order Runge-Kutta method. In the present study, the surface of each capsule was mapped with a minimum of 20x40 elements and a maximum of 40x80 elements. Accuracy of the method was checked by using two different mesh and also by monitoring the volume variation of a capsule. For example with a 20x40 mesh, we find a volume change of less than 0.01% at time kt=40 i.e., after 4000 iterations. Another check of the accuracy of the method is to verify that when the two capsules are very far apart, they reach a steady deformed shape that is identical to the one found for an isolated capsule [4].

RESULTS

We discuss in detail the case of two capsules with a membrane obeying SK law ($K=3G_s$) for $\varepsilon=0.6$. Figure 1 shows the successive profiles of the two capsules as they cross. It should be noted that during the whole process, the flow vorticity keeps the membrane rotating around the internal liquid. During the approach phase (kt=4), C_1 and C_2 have approximately the shape they would have if they were alone in the flow and their centre of mass are still separated by the initial distance $\Delta x_3 = 0.5a$. When they get nearer, their deformation increases substantially, and a peak in deformation energy is observed when the interaction is maximum (kt=8.5). High curvature areas appear on the membrane where folding and damage might occur. During the separation phase (kt=9.7) one of the principal tension become very small near the downstream tip and a tendency towards buckling is observed. Finally as the capsules separate fully, they each recover the shape of an isolated capsule, but the distance Δx_3 has increased from $0.5a$ to $1.2a$. Consequently, the hydrodynamic interaction between the two capsules results in an irreversible shift of trajectory and the x_3 separation increases. This phenomenon was also observed for liquid droplets [5].

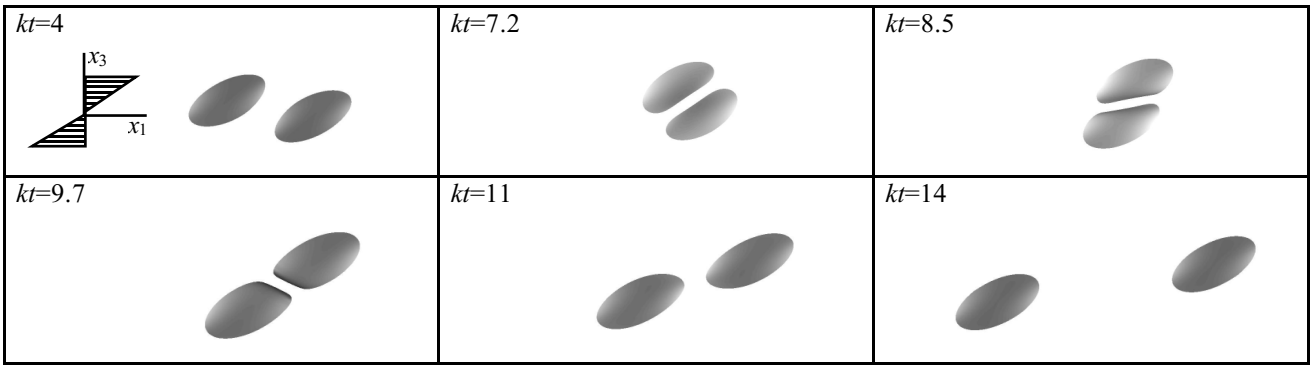

Figure 1. Successive profiles at different time kt, of two capsules for $\varepsilon=0.6$ with a SK membrane. Grey level mapping corresponds to local area dilation. Initial relative position (d_1, d_3)=(-5a, 0.5a).

CONCLUSIONS

There are different interesting features that are specific to capsule interactions. As shown on Figures 1 & 2, when the capsules overpass (Δx_2=0), the centres of mass cross separation Δx_3 is maximum. However, when the capillary number is increased from ε=0.6 to 2.1, the deformation of the capsules is larger, but the final shift in trajectory does not change. The shift seems to depend mainly on initial separation d_3. For the trajectory shift to be negligible, an initial separation $d_3>3a$ is necessary for capsules, whereas $d_3>2a$ is enough for two liquid droplets [5]. The less deformable the capsule, the larger the trajectory shift. Capsules with NH membrane that are strain softening, are easier to deform than capsules with SK membrane, and are thus shifted less. Negative principal tensions appear in the membrane in the wake of separating capsules. Such negative tensions may lead to buckling and burst of the membrane. To avoid this phenomenon, a finite bending resistance of the membrane must be accounted for or the capsule must be pre-stressed by osmotic pressure.

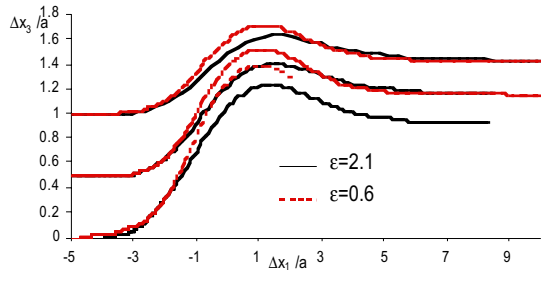

Figure 2. Relative centre of mass trajectories for two capillary numbers

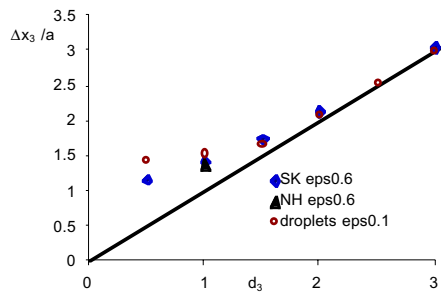

Figure 3 Final shift in trajectory for different membrane laws and initial position

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