APPROXIMATIONS OF STIFFNESS TENSOR OF BONE - DETERMINING AND ACCURACY

J. Piekarski*, K. Kowalczyk-Gajewska*, E. Waarsing**, M. Maździarz*
* *Institute of Fundamental Technological Research, PL Świętokrzyska 21, 00-049 Warsaw, Poland*
** *Erasmus Medical Centre , Orthopaedic Research Laboratory, EE 1614, P.O.BOX 1738, 3000 DR Rotterdam, The Netherlands*

Summary Properties of the apparent stiffness tensor of bone are investigated. The computer reconstruction composed of computer microtomograph imaging, finite element reconstruction and numerical tests results in apparent stiffness tensor of bone. Next the stiffness tensor is subjected to spectral and harmonic decompositions. Kelvin moduli and invariants of orthogonal projectors provided by spectral analysis as well as five invariant parts resulting from harmonic decomposition are obtained. These scalar and tensorial invariants enable the analysis of properties of the stiffness tensor such as material symmetry. Finally, the closest isotropic stiffness tensor is derived and the possible orthotropic approximations are proposed and their accuracy is studied.

INTRODUCTION

The trabecular structure of bone is spatially oriented. Due to the existence of preferred directions apparent mechanical properties of bone commonly are approximated using orthotropic constitutive models. Cowin in [1] has proposed the second order fabric tensor to describe anisotropy of the bone structure that is capable to deal at most with orthotropic material. Another variant of this approach has been considered by Zysset and Curnier [10]. Numerous attempts have been done in order to identify orthotropic properties of bone, for example [9]. Also homogenization methods [8] has been applied to specify orthotropic constitutive models. Nevertheless, it seems that the accuracy of approximation of bone material with orthotropic model have not been investigated quantitatively so far.

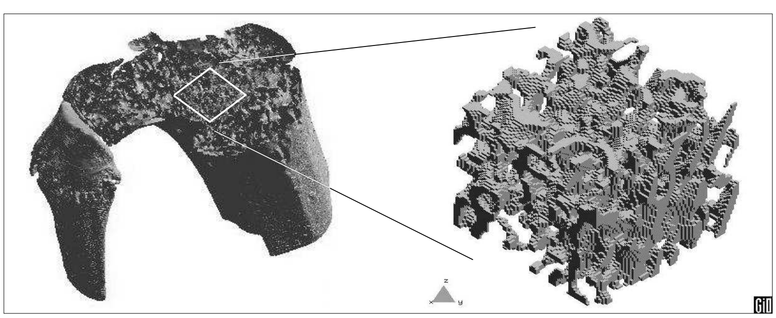
Development of computer microtomography, techniques of reconstruction of bone morphology and assessment of mechanical properties using Finite Element numerical tests provided a significant impact in investigation of constitutive properties of bone tissue. Van Rietbergen, Huiskes at al. [4, 5] showed usefulness of this methodology for quantifying elastic moduli of bone, and Odgaard et al. [3] showed close relation between constitutive tensors evaluated using the computer reconstruction technique and those identified from models based on fabric tensor and mechanical tests.

In this paper we used the computer reconstruction method combined with spectral and harmonic decomposition to investigate properties of the apparent stiffness tensor of bone. Hexahedral samples of trabecular bone was reconstructed from computer microtomograph images, next Finite Element models of samples was build and numerical tests combined with averaging procedure was performed in order to identify the fully anisotropic apparent stiffness tensor. Subsequently, the spectral and harmonic decompositions of the tensor were performed. Six Kelvin moduli and invariants of projectors were evaluated. The closest isotropic tensor and possible orthotropic approximations were specified basing on the harmonic decomposition. Analysis of these quantities allowed to discuss symmetries of apparent stiffness tensor and to measure the deviation of the proposed approximations from the actual stiffness tensor.

COMPUTER RECONSTRUCTION METHOD

The right proximal tibia of rat was scanned using a prototype in-vivo micro-CT scanner (Skyscan 1076, Skyscan, Antwerp, Belgium), resulting in reconstructed data sets with 21.8-micron pixel-size. Data sets were segmented using a local threshold algorithm. Trabecular and cortical bone were separated using EUR in-house software. The example of three-dimensional reconstruction of separated bone structure is presented in Fig.1a. Next, the hexahedral sample was extracted from the reconstructed image, Fig.1b, and the Finite Element (FE) model of the sample was built by replacing each voxel of μCT image with an 8-node brick element. The effective stiffness tensor $\bar{\boldsymbol{C}}$ of the sample was evaluated by applying averaging procedure

$$\bar{\boldsymbol{C}}\bar{\boldsymbol{\varepsilon}} = \frac{1}{V}\int_V \boldsymbol{C}\boldsymbol{\varepsilon}\, dV \qquad (1)$$



a) b)

Fig.1 a) Reconstructed μCT image of bone, b) FE model of hexahedral sample

where $\boldsymbol{\varepsilon}$ denoted strains induced by applying uniform strain state $\bar{\boldsymbol{\varepsilon}}$ at boundaries, $\boldsymbol{C}$ is the local stiffness tensor and V denoted the volume of the sample. In these numerical tests the trabecular tissue was assumed to be uniform and isotropic. The O'Kelly's nanoindentation tests on the unfixed rat trabecular tissue [11] showed the average value of Young modulus equal $8.9\ GPa$ with a 95% confidence interval of $\langle 7, 9.8\rangle\ GPa$, and this average value has been taken as the material constant of bone matrix.

SPECTRAL AND HARMONIC DECOMPOSITION OF THE STIFFNESS TENSOR

An arbitrary 4^{th} rank stiffness tensor $\boldsymbol{C}$ can be presented in the form [6]

$$\boldsymbol{C} = \lambda_I \boldsymbol{w}^I \otimes \boldsymbol{w}^I + \lambda_{II} \boldsymbol{w}^{II} \otimes \boldsymbol{w}^{II} + \ldots + \lambda_{VI} \boldsymbol{w}^{VI} \otimes \boldsymbol{w}^{VI} \tag{2}$$

where $\{\boldsymbol{w}^I,\ \boldsymbol{w}^{II},\ \boldsymbol{w}^{III},\ \boldsymbol{w}^{IV},\ \boldsymbol{w}^{V},\ \boldsymbol{w}^{VI}\}$ constitutes an orthonormal basis in the space of 2^{nd} rank symmetric tensors called elastic eigenstates, and six scalar parameters $\lambda_I,\ \lambda_{II}, \ldots, \lambda_{VI}$ are called Kelvin moduli. The Kelvin moduli and invariants of elastic eigenstates provide 18 independent parameters of the stiffness tensor. By investigating these invariants the structure and properties of the stiffness tensor can be analyzed.

Another tool useful for analysis of the properties of the stiffness tensor $\mathbf{C}$ is the harmonic decomposition [2], [7]. In view of this decomposition the tensor $\boldsymbol{C}$ is uniquely defined by the set of parameters: $\boldsymbol{C} \Longleftrightarrow (\boldsymbol{h_P},\ \ \boldsymbol{h_D},\ \ \boldsymbol{\varphi},\ \ \boldsymbol{\rho},\ \ \boldsymbol{D})$ among which h_P and h_D are scalars, $\boldsymbol{\varphi}$ and ρ are symmetric second rank deviators and $\boldsymbol{D}$ is totally symmetric and traceless 4^{th} rank tensor called the 4^{th} rank deviator. The particular form and properties of the tensorial parameters give further insight into material symmetry analysis. Moreover, harmonic decomposition enables finding out the closest isotropic approximation of $\boldsymbol{C}$ and provides hints for orthotropic approximation of $\boldsymbol{C}$.

ANALYSIS OF ACTUAL EFFECTIVE STIFFNESS TENSOR OF TRABECULAR BONE

The sample of rat trabecular bone of the edge size 1.75 *mm* has been treated with the procedure described in previous sections. The effective stiffness tensor $\bar{\boldsymbol{C}}$ has been numerically identified. The tensor is characterized by the following Kelvin moduli: $\{252.54,\ 282.81,\ 373.79,\ 421.81,\ 451.34,\ 902.31\}$ $[MPa]$. Nevertheless only one eigenstate related to the third Kelvin modulus is close to a deviator, therefore it is hard to specify significant symmetries of the tensor without additional analysis [2].

Applying the harmonic decomposition one obtains $h_P = 790.33\ MPa,\ \ h_D = 378.85\ MPa$. These values provide the closest isotropic approximation with the Young modulus $E = 458.41$ *MPa* and the Poisson ratio $\nu = 0.210$. The relative difference between the closest isotropic tensor $\bar{\boldsymbol{C}}^{is}$ and the actual tensor $\bar{\boldsymbol{C}}$ measured according to $d = \{[(\bar{\boldsymbol{C}}^{is} - \bar{\boldsymbol{C}}) : (\bar{\boldsymbol{C}}^{is} - \bar{\boldsymbol{C}})]/(\bar{\boldsymbol{C}} : \bar{\boldsymbol{C}})\}^{1/2}$ (see [7]) equals 0.304. Second order deviators $\boldsymbol{\varphi}$ and ρ are almost coaxial suggesting position of approximated orthotropy axes. The orthotropic approximation $\bar{\boldsymbol{C}}^{orth}$, obtained by rotating the tensor to axes specified from harmonic decomposition and neglecting out-of-orthotropy terms in the rotated representation, is characterized by the following Kelvin moduli: $\{262.54,\ 281.58 = G_{12},\ 370.39 = G_{23},\ 427.95,\ 440.48 = G_{13},\ \ 901.45\}$ $[MPa]$ with the Young moduli $E_1 = 435.67$ *MPa*, $E_2 = 282.51$ *MPa*, $E_3 = 705.76$ *MPa*, Poisson ratios $\nu_{12} = 0.252,\ \nu_{13} = 0.161,\ \nu_{23} = 0.131$, and the norm of relative distance to the actual tensor $d = 0.052$.

CONCLUDING REMARKS

The aim of the paper was to discuss the method of determining the isotropic and the orthotropic approximation of actual effective stiffness tensor of trabecular bone and to estimate errors of these approximations. The method is based on both spectral and harmonic decomposition of 4^{th} rank stiffness tensor. The analysis of an actual effective stiffness tensor obtained by means of computer reconstruction method is provided in order to illustrate the proposed procedure. In this particular case the orthotropic approximation appears to be very good in spite of lack of evident symmetries. Nevertheless, general conclusions could be formulated on the basis of more waste set of data. Such work is in progress.

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