

# ACTIVE CONTROL OF FGM PLATES USING DISTRIBUTED PIEZOELECTRIC SENSORS AND ACTUATORS

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**Summary** Distributed sensing and active control of FGM plates using piezoelectric sensors/actuators is studied. A nine noded piezolaminated plate finite element has been developed incorporating the FGM material model and the electromechanical coupling constitutive relations of the piezoelectric sensors/actuators. The FGM plate considered for study is made of the combined aluminum oxide and a titanium alloy, Ti-6Al-4V and its properties are graded through the thickness according to a volume fraction power law distribution. The vibration control performance is explored using the LQR optimal control law. The influence of the constituent volume fraction of Ti-6Al-4V is also studied for the static and controlled dynamic response of FGM plates.

## INTRODUCTION AND FORMULATION

“Functionally graded materials” (FGMs) are relatively new class of composite materials which are characterized by the smooth and continuous variation of the mechanical properties from one surface to the other. Due to its superior thermo-mechanical properties, FGMs have found extensive applications in aerospace, automobile, biomedical and nuclear industries. A FGM plate made of two materials whose properties are graded through the thickness direction according to volume fraction power law, is considered with thin PZT layers embedded on top and bottom surfaces to act as distributed sensor/actuator. The material properties can be expressed as,  $P_{eff}(z) = P_1 V_1(z) + P_2 (1 - V_1(z))$ , where  $P_{eff}$  is the effective material property of the FGM, and  $P_1$  and  $P_2$  are the properties of the materials 1 and 2 respectively.  $V_1$  is the volume fraction of the constituent material 1 in the FGM and can be written as,  $V_1 = \left(\frac{2z+h}{2h}\right)^n$  where  $n$  is the

volume fraction exponent ( $0 \leq n \leq \infty$ ). A  $C^\infty$  continuous, shear flexible, nine-noded piezolaminated plate finite element derived including electromechanical coupling effects of the piezoelectric sensor/actuator layers [1] has been used for the study. The element has been further developed to include the FGM material model as above. The piezoelectric constitutive equations coupling elastic and electric fields are expressed as,  $\{\sigma\} = [c] \{\epsilon\} - [e]^T \{E\}$  and  $\{D\} = [e] \{\epsilon\} + [\epsilon] \{E\}$ , where  $\{D\}$ ,  $\{E\}$ ,  $\{\epsilon\}$  and  $\{\sigma\}$  are electrical displacement, electrical field, strain and stress vectors, while,  $[C]$ ,  $[e]$  and  $[\epsilon]$ , are elasticity, piezoelectric and dielectric constant matrices. The displacement field, strain definitions, electric field strength definitions and shape functions are as detailed in Ref.[1]. The element has five elastic degrees of freedom,  $u_o, v_o, w$  which are displacements and  $\theta_x, \theta_y$  which are rotations of normal in  $xz$  &  $yz$  planes and one electrical degree-of-freedom,  $\phi$  per piezoelectric layer. Using the variational principles the strain energy functional is given by,

$$U = \frac{1}{2} \int_A \left[ \{\epsilon_p\}^T [A_{ij}] \{\epsilon_p\} + \{\epsilon_b\}^T [B_{ij}] \{\epsilon_b\} + \{\epsilon_s\}^T [D_{ij}] \{\epsilon_s\} + \{\epsilon_s\}^T [E_{kl}] \{\epsilon_s\} \right] dA - \int_{V_p} \left( \{\epsilon_p\}^T + z \{\epsilon_b\}^T \right) [e]^T \{E\} dV$$

where,  $\{\epsilon_p\}$ ,  $\{\epsilon_b\}$  and  $\{\epsilon_s\}$  are the inplane, bending & shear strains.  $[A_{ij}]$ ,  $[B_{ij}]$ ,  $[D_{ij}]$ , ( $i, j = 1, 2, 6$ ) &  $[E_{kl}]$  ( $k, l = 4, 5$ ) are extensional, bending-extensional, bending and shear stiffness coefficients of the plate. These are given by,

$$[A_{ij}], [B_{ij}], [D_{ij}] = \int_{-h/2}^{h/2} \left( ([Q_{ij}^1] - [Q_{ij}^2]) \left( \frac{2z+h}{2h} \right)^n (1, z, z^2) + [Q_{ij}^2] (1, z, z^2) \right) dz \quad \& \quad [E_{kl}] = \int_{-h/2}^{h/2} \left( ([Q_{kl}^1] - [Q_{kl}^2]) \left( \frac{2z+h}{2h} \right)^n (1, z, z^2) + [Q_{kl}^2] (1, z, z^2) \right) dz$$

where  $[Q^1]$  &  $[Q^2]$  are the elastic properties. The electrical energy is as in [1] while the kinetic energy is given by,

$$T = \frac{1}{2} \int_A \left[ P(\dot{u}^2 + \dot{v}^2 + \dot{w}^2) + I(\dot{\theta}_x^2 + \dot{\theta}_y^2) \right] dA, \quad \text{where, } [P, I] = \sum_{k=1}^m \int_{-h_{k-1}}^{h_k} \left( (\rho^1 - \rho^2) \left( \frac{2z+h}{2h} \right)^n (1, z^2) + \rho^2 (1, z^2) \right) dz$$

where  $m$  is the number of layers. The Langrange's equations of motion, for an element can be written as

$[M_{uu}]_e \{\ddot{\delta}\}_e + [K_{uu}]_e \{\delta\}_e + [K_{u\phi}]_e \{\phi\}_e = \{F_s\}_e$  and  $[K_{\phi u}]_e \{\delta\}_e + [K_{\phi\phi}]_e \{\phi\}_e = \{F_q\}_e$ , where,  $[M_{uu}]_e$  is element mass matrix and  $[K_{uu}]_e$  is element elastic stiffness matrix obtained from the kinetic and strain energy functionals [1].  $[K_{\phi u}]_e = [K_{u\phi}]_e^T$  are the elastic-electric coupling stiffness matrices and  $[K_{\phi\phi}]_e$  is the electric stiffness matrix,  $\{F_s\}_e$  is the applied mechanical force and  $\{F_q\}_e$  is the applied electrical charge. The sensor voltage is given by,  $\{\phi\}_e = -[K_{\phi\phi}]_e^{-1} [K_{\phi u}]_e \{\delta\}_e$ .

The assembled global equations are converted into state space form and active control performance is studied using LQR optimal control scheme [1]. The FGM plate considered for study is of aluminum oxide and titanium alloy, Ti-6Al-4V and its properties are graded through thickness according to a volume fraction power law. The size of the plate is 400 x 400mm with 5mm thickness and is in the cantilevered configuration with one edge fixed and others free. The PZT sensors/actuators covering the top and bottom surfaces of the FGM plate have a thickness of 0.1mm. The material properties are given in Table 1.

**Table 1 : Material properties**

| Material       | E (N/m <sup>2</sup> )   | N      | $\rho$ (kg/m <sup>3</sup> ) | $d_{31}/d_{32}$ (m/V) | $\epsilon_{33}$ (F/m) |
|----------------|-------------------------|--------|-----------------------------|-----------------------|-----------------------|
| Aluminum oxide | $3.2024 \times 10^{11}$ | 0.26   | 3750                        | -                     | -                     |
| Ti-6Al-4V      | $1.057 \times 10^{11}$  | 0.2981 | 4429                        | -                     | -                     |
| PZT G1195N     | $63 \times 10^9$        | 0.3    | 7600                        | $254 \times 10^{-12}$ | $15 \times 10^{-9}$   |

## RESULTS AND DISCUSSION

The cantilevered FGM plate is subjected to uniform pressure of  $100 \text{ N/m}^2$  and the centerline deflection for various values of the constituent volume fraction  $n$  is as shown in figure 1. As  $n$  increases, the volume fraction of Ti-6Al-4V is decreased and when  $n$  tends to  $\infty$ , the FGM plate almost totally consists of aluminum oxide. As  $n$  increases, the deflection of FGM plate is decreased. To control the plate deflection, active voltages of equal magnitude and opposite sign are applied across the PZT layers, through the thickness. Figure 2 shows the centerline deflection for various values of the power law exponent  $n$  under actuator voltage  $V = 40 \text{ V}$ . By adjusting the actuator voltage, one can control shape as well as position of maximum deflection point. These results compare well with ref [2]. Then six natural frequencies of the FGM plate as function of volume fraction power law exponent  $n$  are compared with ref [2] as in table 2. The for active vibration control, the top and bottom PZT layers are used as an integrated sensor and actuators. An impact load of  $50 \text{ N}$  is applied at the free edge of the plate for a duration of  $1 \text{ msec}$ . The LQR optimal control scheme is applied to study the active vibration control with the weighting parameters  $Q = 1 \times 10^8$  and  $R = 1$ . An initial modal damping of  $0.2\%$  is assumed and the controller is activated after  $0.2 \text{ sec}$  of the impact. Figure 3 indicates the controlled dynamic response of the FGM plate for the volume fraction power law exponent  $n$  of  $0.5$  and  $1000$ . The effect of  $n$  in this case is similar to that of the static case. i.e., the amplitude decreases with the increase in  $n$ .

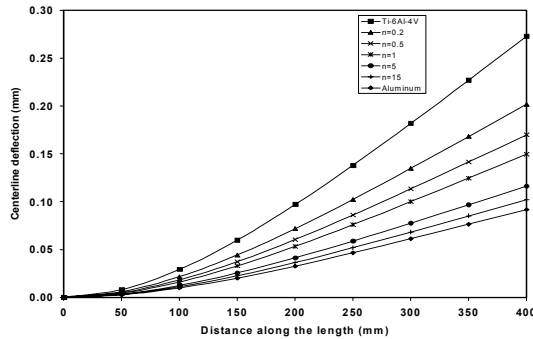


Fig. 1 : Centerline deflection of the FGM plate under uniformly distributed load of  $100 \text{ N/m}^2$  and actuator voltage of  $0 \text{ V}$ .

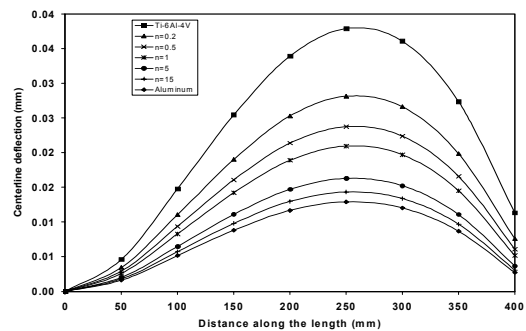


Fig. 2 : Centerline deflection of the FGM plate under uniformly distributed load of  $100 \text{ N/m}^2$  and actuator voltage of  $40 \text{ V}$ .

| Mode No. | n=0     |        | n=0.5   |         | n=15    |         | n=1000  |     |
|----------|---------|--------|---------|---------|---------|---------|---------|-----|
|          | present | [2]    | present | present | present | present | present | [2] |
| 1        | 25.51   | 25.58  | 33.224  | 45.080  | 47.806  | 46.55   |         |     |
| 2        | 62.49   | 62.75  | 82.053  | 112.13  | 119.01  | 116.00  |         |     |
| 3        | 156.73  | 157.20 | 204.77  | 278.56  | 295.49  | 287.60  |         |     |
| 4        | 199.67  | 200.19 | 260.31  | 353.54  | 374.96  | 365.00  |         |     |
| 5        | 227.93  | 228.22 | 298.47  | 406.91  | 431.72  | 419.55  |         |     |
| 6        | 396.54  | 397.58 | 519.21  | 707.75  | 750.91  | 730.59  |         |     |

Table 2 : Natural frequencies of the cantilevered FGM plate.

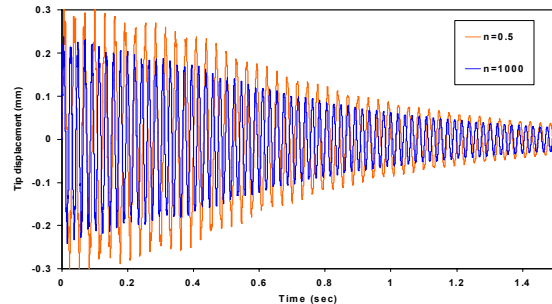


Fig. 3 : Controlled dynamic response of the FGM plate under an edge load of  $50 \text{ N}$  for  $1 \text{ msec}$  and LQR control ( $Q=1e8$ ,  $R=1$ )

## CONCLUSION

In this work a shear deformable piezolaminated FGM plate finite element has been developed and shape and vibration control performance of an FGM plate is studied using distributed piezoelectric sensors and actuators. The properties of the FGM plate are functionally graded through the thickness direction according to a volume fraction power law distribution. Optimal control theory LQR scheme has been applied for the active control.

## References

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- [2] He, X.Q., Ng, T.Y., Sivashanker, S., Liew, K.M.: Active control of FGM plates with integrated piezoelectric sensors and actuators, *Int. JI. Of Sol. Struct.*, 38, 2001, 1641-1655.