ON THE NONLINEAR DYNAMICS OF MULTICOMPONENT DYNAMICAL SYSTEMS

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Summary We propose a framework for analysis of dynamical systems with a large number of heterogeneous coupled components. This framework is used to propose a graph-theoretic decomposition that exhibits a chain of vertically unidirectionaly coupled levels and horizontally decoupled components within each vertical level. The decomposition is used to prove a number of results on asymptotic dynamics of coupled dynamical systems.

INTRODUCTION

A number of applications of current interest, such as large-scale engineered systems and genetic regulatory networks involve a large number of heterogeneous, connected components, whose dynamics is affected by possibly an equally large number of parameters. The classical dynamical systems approach attempts to analyze such systems via studying their trajectories in their high-dimensional phase space. However, dynamical systems techniques are mostly developed for low-dimensional systems. We introduce here a framework for studying such multicomponent systems and a graph-theoretic decomposition that for a particular class of networks allows for efficient application of methods of low-dimensional dynamical systems. We also present examples of such decomposition for some high-dimensional systems.

HORIZONTAL-VERTICAL DECOMPOSITION

The following result on decomposition of systems of ordinary differential equations can be proven using a graph-theoretic approach:

Theorem 1 *(Horizontal-vertical decomposition) Any system of ordinary differential equations can be decomposed into* k *vertical levels, such that each higher level is driven by the dynamics of levels below. Every horizontal level* $i \subset \{1, ..., k\}$ *can be decomposed into* m_i *sets, which have dynamics independent of each other.*

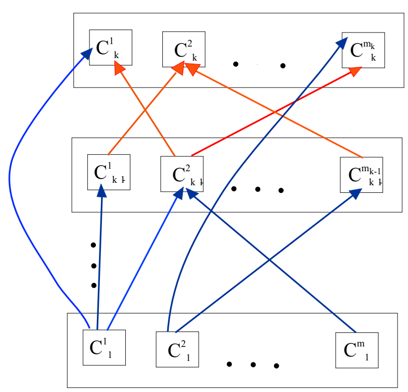

Figure 1. Decomposition described in the theorem

The states at the lowest level in some sense "control" the behavior of all the others. This representation of the system allows for analysis using low-dimensional dynamical systems methods, as we describe next.

APPLICATIONS

The decomposition above is quite useful for analyzing asymptotic dynamics of a variety of coupled systems. The simplest result in this direction is:

Proposition 2 *Assume every C_i^j contains only one state and the dynamics of each is bounded. Then the system asymptotes to a fixed point that is not at infinity.*

Proof. If each of the systems at the first level is a one-dimensional system asymptotically converging to a fixed point, its "input" into the second and higher levels is asymptotically constant in time. But, since each of the subsystems C_i^j has bounded, one-dimensional dynamics, this means that each C_i^j will converge to a fixed point. ■

A fuel cell model

Here we show an application of the above decomposition to the 10-state fuel-cell dynamics model discussed in [1]. The system has 10 components, states of which are defined on $\mathbb{R}$, that we label by the index set $\mathcal{N} = \{1, ...10\}$. We obtain decomposition shown in figure 2. Every variable affects itself, but self-loops are not shown in the figure for clarity. Level

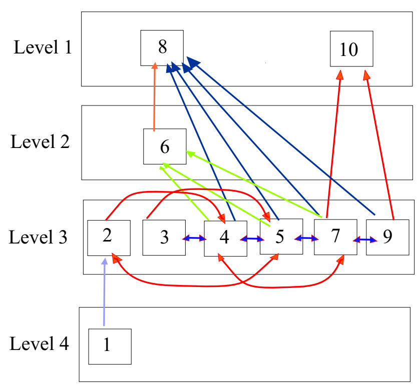

Figure 2. Decomposition for the fuel cell problem.

4 of the decomposition contains only state 1. The dynamics of this state thus tends to a fixed point and asymptotically can be taken just as a parameter input into level 3, where it enters through dynamics of state 2. Note that Level 3 of the decomposition contains six states that have various feedback loops. Level 3 in turn affects states 6, 8, 9, but each of these is a one dimensional state at either level 2 or level one so it is easy to prove that

Proposition 3 *If the subsystem $\{2, 3, 4, 5, 7, 9\}$ tends to a fixed point, then the system tends to a fixed point. If the subsystem $\{2, 3, 4, 5, 7, 9\}$ tends to a limit cycle, then the system tends to a limit cycle.*

Proof. All of the systems at levels higher than 2 are 1-dimensional. If input to these is asymptotically constant, these subsystems will tend to a fixed point. If the input is time-periodic with a fixed period T, these will oscillate in time with the same period as the input. ■

CONCLUSIONS

Graph-theoretic methods lead to a decomposition fo a system of ordinary differential equations into vertical levels. At each vertical level there are subcomponents that do not interact with each other. Each vertical level only affects levels above it, and is affected by states at the same level and below. The method of decomposition is useful for showing that large classes of possibly very high dimensional networked systems can possess low dimensional (periodic, quasi-periodic, low-dimensional chaotic) dynamics. We presented an application to a 10-state model of fuel cell dynamics.

References

[1] Pukrushpan, J., Stefanopolou, A., Varigonda, S., Eborn, J. and Haughtetter, C.: Control-Oriented Model of Fuel Processor for Hydrogen Generation in Fuel Cell Applications,*Submitted*,2003.