NONLINEAR DYNAMICS OF HIGH-SPEED MILLING

Gabor Stepan, Tamas Insperger, Robert Szalai

Department of Applied Mechanics, Budapest University of Technology and Economics

Budapest, H-1111, Hungary

Summary In case of highly interrupted machining, the ratio of time spent cutting to not cutting is considered as a small parameter, and the classical regenerative vibration model breaks down to a simplified discrete mathematical model. The linear analysis of this discrete model leads to the recognition of the doubling of instability lobes in the stability charts of machining parameters. This kind of lobe doubling is related to the appearance of period doubling bifurcations occurring primarily in low-immersion high-speed milling along with the classical self-excited vibrations (or secondary Hopf bifurcations). The present work investigates the nonlinear vibrations in case of period doubling and compares this to the well-known subcritical nature of the Hopf bifurcations in turning processes. Our experimental results draw the attention to the limitations on the highly interrupted cutting condition. The analysis of the general milling model requires the use of the stability chart of the delayed Mathieu equation, which is also constructed.

INTRODUCTION AND MODELLING

High-speed milling is one of the most preferred and efficient cutting processes nowadays. It is a challenging task for researchers to explore its special dynamical properties, including the stability conditions of the cutting process and the nonlinear vibrations that may occur near to the stability boundaries. These dynamical properties are mainly related to the underlying regenerative effect in the same way as it is in case of the classical turning process. Still, some new phenomena appear for low-immersion milling as predicted in [2,3]. These phenomena were also reported in [4,5] in case of milling, independently from the immersion or speed characteristics of the milling processes.

High-speed milling often means also low immersion (very small chip thickness) with relatively small number (z = 2 to 4) of cutting edges on the tool (see Figure 1). There is either no contact between the tool and the work-piece, or there is only one edge in contact for a (relatively short) time. In these cases, highly interrupted machining can well approximate the whole machining process. In the simplest models of highly interrupted machining, the ratio of time spent cutting to not cutting is a small parameter. This leads to a mechanical model where the free vibration of the tool is perturbed periodically by an impact to the work-piece. This impact results a sudden change in the tool oscillation velocity and it depends on the actual chip thickness, that is, on the difference of the present and the previous tool position. In this respect, the model includes the classical regenerative effect described for turning in [1]. There is one major difference, though: the periods of no-contact are regulated by the cutting speed parameter. Accordingly, high-speed milling is a kind of parametrically interrupted cutting as opposed to the self-interrupted cutting arising in unstable turning processes.

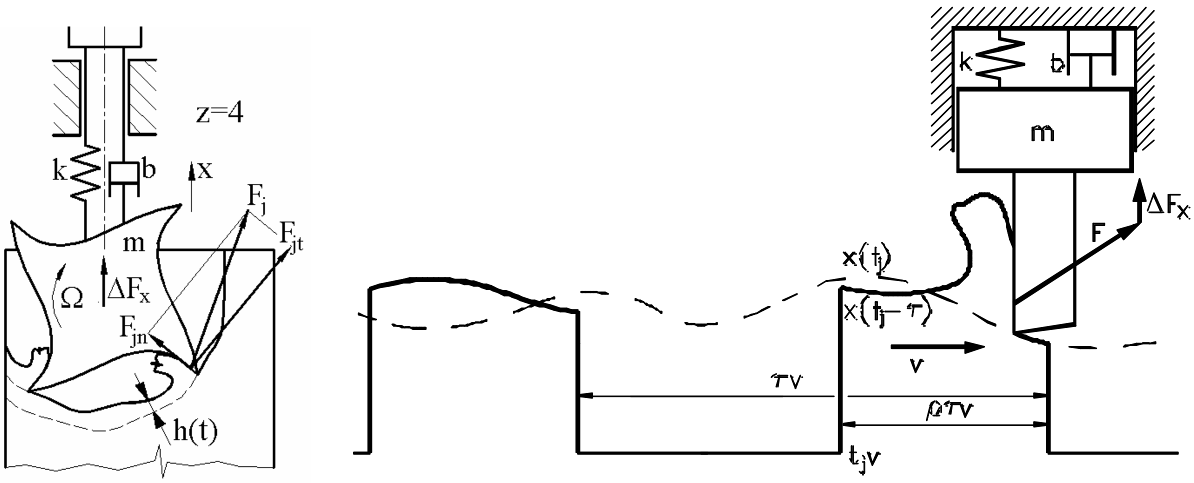

Fig. 1. Mechanical models of high-speed milling for normal and highly interrupted cutting cases

STABILITY AND NONLINEAR VIBRATIONS OF HIGHLY INTERRUPTED CUTTING

The simplest possible, but still nonlinear highly interrupted cutting model leads to a two-dimensional discrete mathematical model. Bifurcation analysis can be carried out along the stability limits related to period doubling bifurcations and (secondary) Hopf bifurcations. These require centre manifold reduction and normal form transformation. The tedious algebraic work can be carried out in closed form and leads to nonlinear phenomena similar to the one experienced in the case of the Hopf bifurcation in the turning process. The resulting subcritical bifurcations are presented in analytical form in the subsequent Sections. The existence of stable period two vibrations is also shown 'outside' the unstable period two vibrations. The stable ones can, however, quickly bifurcate to chaotic oscillations with increasing chip width. This is also shown by numerical investigation, and some analytical explanation is also given.

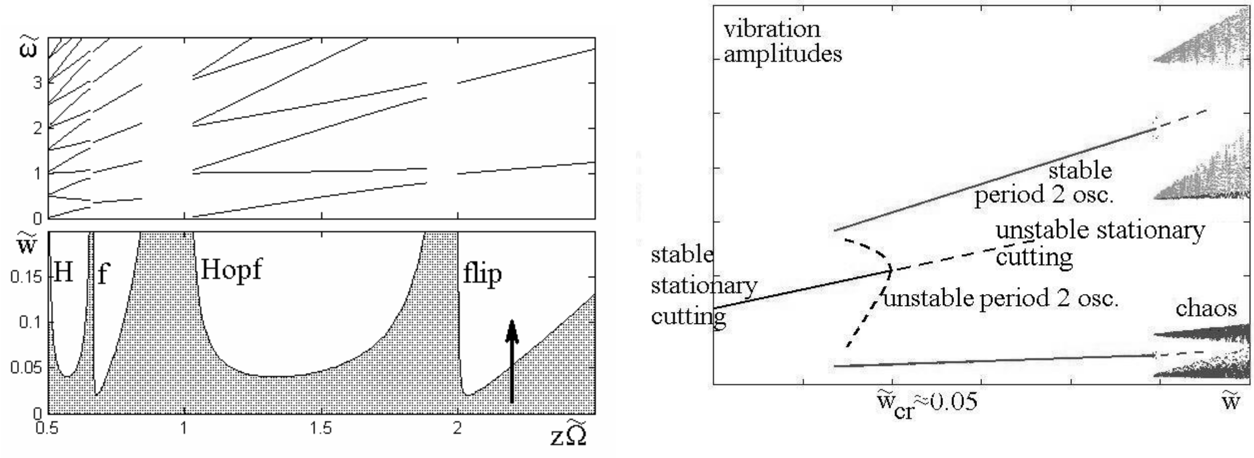

Fig. 2. Stability chart and bifurcation diagram for highly interrupted cutting

The stability chart of Fig. 2 is presented in the plane of the dimensionless cutting speed and chip width parameters, and it also presents the dimensionless vibration frequencies along the stability limits above the stability chart. This chart and the corresponding spectra are all confirmed experimentally, too. Figure 2 also presents the bifurcation diagram at the black arrow crossing the stability boundary at a flip (period doubling) bifurcation in the stability chart.

STABILITY OF MILLING

Experiments drew the attention to the weakness of the small parameter condition on the ratio of time spent cutting to not cutting. If this condition is not applicable, like in the case of full-immersion milling, the mathematical model cannot be simplified to a discrete one, and its structure remains similar to the delayed Mathieu equation:

$$\ddot{x}(t) + (d + e\cos t)x(t) = b\,x(t-2p)\ ,$$

where the time-periodicity and the time delay are equal (here, the dimensionless $2p$) and they are both related to the tooth-pass period (as shown in Fig. 1). In Figure 3, the corresponding stability chart in the space of the three parameters inherits the structure of the Incze-Strutt and the Hsu-Bhatt charts, too [6].

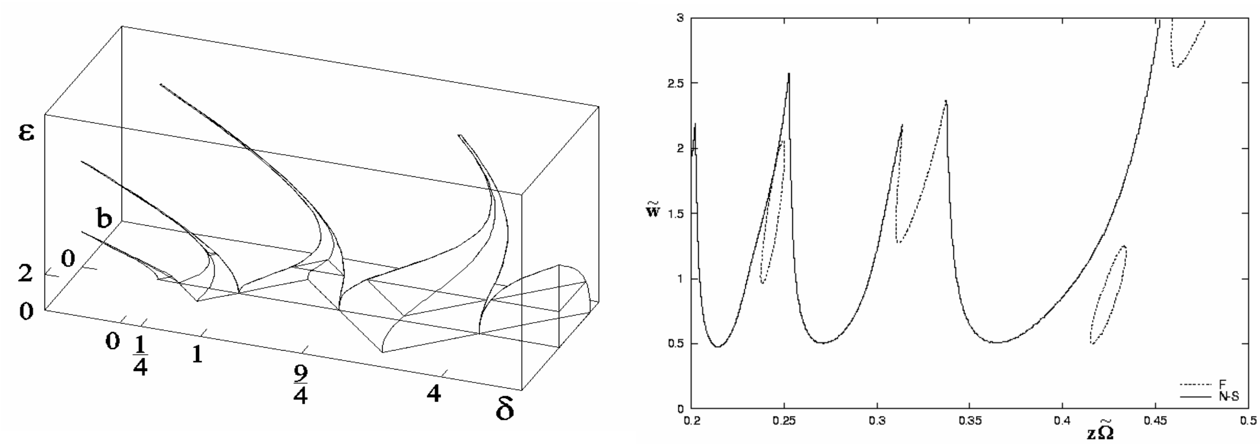

Fig. 3. The stability charts of the delayed Mathieu equation and the corresponding milling process

The construction of the corresponding stability charts in the plane of the same milling parameters as before shows, that the instability lobes related to Hopf bifurcations are actually extended by instability lenses related to the flip or period doubling bifurcations.

References

[1] G. Stépán, Philosophical Transactions of the Royal Society 359, 739-757 (2001).
[2] M.A. Davies, J.R. Pratt, B. Dutterer, T.J. Burns, Journal of Manufacturing Science and Engineering 124(2), 217-225 (2002).
[3] Bayly, P. V., Halley, J. E., Mann, B. P., Davies, M. A., *Journal of Manufacturing Science and Engineering*, 125, 220-225 (2003).
[4] Insperger, T., Stépán, G., *Journal of Manufacturing Science and Engineering*, in press (2004).
[5] B. Balachandran, Philosophical Transactions of the Royal Society 359, 793-820 (2001).
[6] Insperger, T., Stépán, G., *Proceedings of The Royal Society, Mathematical, Physical and Engineering Sciences*, 458, 1989-1998 (2002).