NONLINEAR DYNAMICS OF PARAMETRICAL EXCITED TWO-DEGREES-OF-FREEDOM FREEDOM FLEXIBLE PENDULUM

Amir Abbas Zadpoor*
*Department of Mechanical Engineering, Iran University of Science and Technology, Narmak, Tehran 16844, Iran

Summary This paper explores dynamical behaviour of a parametrical excited two-degree-of-freedom pendulum. It has been shown that as excitation period increases, existing period orbit could bifurcates into more-complex invariant manifolds. Complex behaviour of the system is studied by the means of phase portraits, Poincare sections, and Lyapunov exponents and it has concluded that system could become chaotic for some range of involving parameters.

INTRODUCTION

Bayly and Virgin discussed the stability of forced radial oscillations of a flexible pendulum [1]. Zaki *et al* discussed the effect of stiffness nonlinearity on dynamics of the tangential forced flexible pendulum [2] and shown that alternation of system's dynamic is explored when amplitude of the harmonic force is varied. It has been shown that transition of the periodic motion to chaotic one begins with a Hopf bifurcation and follows by a torus doubling. Bifurcation ends by a torus breaking. The present study considers a flexible pendulum with a vertically moving pivot. The vertical motion is assumed to be harmonic. Due to vertical displacement, the system becomes parametrically excited. Stiffness of the spring is assumed linear and dynamics of the system is studied when amplitude of the pivot's vertical motion is varied. Then the linear spring is replaced with weakly nonlinear and strongly nonlinear ones. Dynamical behavior in two latter cases is compared with each other and with the linear stiffened case. Routes to chaos are studied through construction of the phase portraits and Poincare maps and computation of Lyapunov exponents. A schematic of the problem is given in Figure 1.

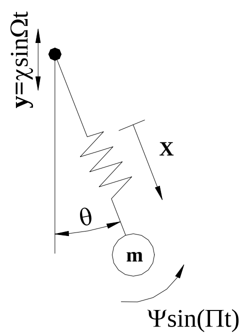

Figure 1 Schematics of the Problem

PROBLEM FORMULATION

Two generalized coordinates, say X and θ, are chosen to derive the governing system of nonlinear ordinary differential equations. The spring, in the general case, assumed cubic. Therefore, the spring force can be represented as

$$F = -kx - \varepsilon x^3$$

Calculating the terms of kinetic and potential energy as well as non-conservative generalized forces and putting them into Lagrange equation leads to the governing system of two nonlinear differential equations. Introducing some new variables and doing some algebra, the non-dimensional form of the governing equation could be written as

$$\ddot{x} + x + \lambda x^3 - [\alpha \sin \Sigma\tau + \beta]\cos\theta - (1+x)\dot{\theta}^2 = 0$$

$$\ddot{\theta}(1+x)^2 + \zeta\dot{\theta} + [\alpha \sin \Sigma\tau + \beta](1+x)\sin\theta + 2(1+x)\dot{x}\dot{\theta} = \psi \sin \Lambda\tau$$

where

$$X = xl \qquad t = \tau\omega \qquad \omega = \sqrt{\frac{k}{m}} \qquad ' = \frac{d}{dt} = \omega\frac{d}{d\tau}$$

$$\tau\Sigma = t\Omega \qquad \lambda = \frac{\varepsilon l^2}{k} \qquad \alpha = \frac{\Omega^2\chi}{l\omega^2} \qquad \beta = \frac{g}{l\omega^2} \qquad \zeta = \frac{c}{m\omega l^2} \qquad \tau\Lambda = t\Pi \qquad \psi = \frac{\Psi}{l^2\omega^2}$$

DYNAMICAL EXPLORATION

An analogy is constructed between rotational and longitudinal motions in one side and ship's rolling angle and ship's pitch on the other side. Current study improves the model to a parametrical excited one, which approximates the reality much closer. Bifurcation study of the system begins from a periodic orbit. Amplitude of the parametric excitation is used as control parameter and alternation of system's dynamics is explored when the control parameter varies. Non-dimensional form of governing equations is digitally simulated using a Runge-Kutta algorithm. Phase portrait in conjunction with Poincare section are used for dynamical study of the system. Poincare sections are constructed in a manner to give a stroboscopic picture of the system's trajectory. Therefore the point in which trajectory cross the Poincare section is recorded for a fixed phase angle of harmonic pivot's excitation. When the system seems to be chaotic, it becomes necessary to use chaos identification methods to realize if it is a true chaotic motion. Lyapunov exponents who are a measure of divergence of nearby trajectories are used for quantification of the system's dependence on initial conditions.

Figure 1 depicts phase portrait of the periodic orbit. The parameters used for integration of the system are given in the Table 1. As excitation amplitude increases, dynamical behavior of the system changes. Figure 3, 4 and 5 show the chaotic behavior of the system. Lyapunov exponents are used for identifying the chaos. The computed values are also listed in Table 1.

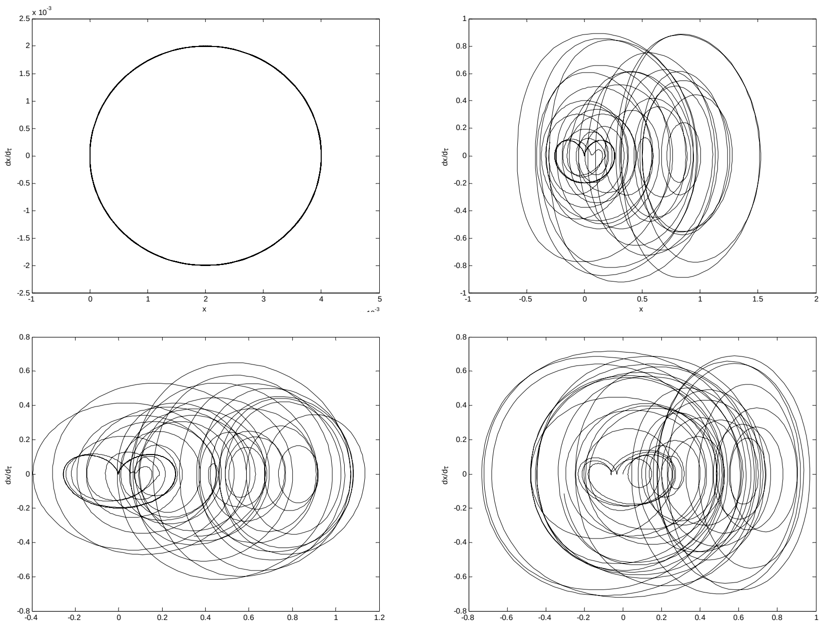

Figure 2. Above Left **Figure 3. Above Right** **Figure 4. Below Left** **Figure 5. Below Right**

	α	β	λ	Σ	ζ	ψ	Λ	**Lyapunov Exponent**
Figure 2	1e-6	0.002	0	0.5	0.005	0.0001	0.3	-2.43657e-9
Figure 3	0.15	0.002	0	0.5	0.005	0.0001	0.3	0.0596394
Figure 4	0.15	0.002	0.05	0.5	0.005	0.0001	0.3	0.0199838
Figure 5	0.15	0.002	0.9	0.5	0.005	0.0001	0.3	0.0666035

Table 1

CONCLUSIONS

It has been shown that the system becomes chaotic for some range of involving parameters. Computed Lyapunov exponents and phase portraits are used to demonstrate complexity of the involving system. Due to space constraints, Poincare sections and phase portraits, which are used for bifurcation study of the system, are omitted in this extended summary.

References

[1] Bayly, P. V., and Virgin L. N.: On the Forced Radial Oscillations of an Experimental Spring Pendulum, *Proceedings of the 14th Biennial ASME conference on Mechanical Vibrations and Noise*, 1993, pp. 229-236

[2] Zaki, K., Rajagopal, K. R., and Srinivasa, A. R.: Effect of Nonlinear Stiffness on the Motion of a Flexible Pendulum, *Nonlinear Dynamics*, **27**:1-18, 2002