FREQUENCY- AND DIRECTIONALITY-CONTINUATION SCHEMES FOR SCATTERER SHAPE DETECTION IN ACOUSTICS

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Summary In this paper we discuss continuation schemes for detecting the location and shape of rigid scatterers embedded in a host acoustic medium when considering scant measurements of the scattered acoustic pressure in the region exterior to the scatterer (near- or far-field). Underlying the inversion process are frequency- and directionality-continuation iterative schemes that allow robust determination of the unknown shape using a small number of frequencies and directions of the probing insonifying waves. The methodology is based on boundary integral equation for the solution of the forward problem. The scattered pressure is measured only in the backscatter region and at receiver locations that do not circumscribe the sought scatterer. Several numerical results are presented, both for parametrically defined shapes (e.g. circle, ellipse, etc), as well as for penny-shaped scatterers and completely arbitrary geometries. For the latter cases, conditions for non-self-intersecting shapes complement the numerical implementation to allow for the recovery of the nodal coordinates of the meshed scatterer boundary.

INTRODUCTION

Inverse scattering problems are of considerable practical interest in various areas of science and engineering due, in part, to the ever broadening spectrum of important applications that range from medical, to geophysical, to target identification investigations. In particular, inverse problems arising in acoustics are of relevance in, amongst others, ultrasound imaging (for medical or other non-destructive assessments), seismic imaging, underwater surveillance and target acquisition, and in the detection of objects in the ocean, either fully submerged or partially buried in the seafloor. Various approaches to resolve the unknown scatterer from the scant measurements are investigated among many researchers [1, 2, 3] and one may roughly classify them into methods that rely on optimization-based schemes [2] and methods that do not explicitly seek to minimize a misfit functional [3]. The advantage of the latter category is that the shape reconstruction can be carried out without a-priori information, whereas, when optimization methods are used, the feasibility space may be considerably narrow. In this work we favor optimization methods for the generality they offer and report on a dual frequency- and (optionally) directionality-continuation algorithm that has, thus far, provided promising results.

THE FORWARD PROBLEM

We consider sound-hard scatterers in 2D full-space (Ω) insonified by traveling plane waves. The forward problem is described by the Helmholtz equation for the scattered pressure u^s, subject to a Neumann condition on the scatterer's surface (Γ), and the Sommerfeld radiation condition at infinity. We use a standard integral equation formulation to arrive at a solution of the forward problem at each iteration of the inverse problem. Using customary notation for denoting the single-layers ($\mathfrak{S}$ and S), the double-layers ($\mathfrak{D}$ and D), and the normal derivative (u^s_ν), the scattered pressure is given as:

$$u^s = \mathfrak{S}[u^s_\nu] - \mathfrak{D}[u^s], \text{ in } \Omega, \quad \text{and} \quad \frac{1}{2}u^s - S[u^s_\nu] + D[u^s] = 0, \text{ on } \Gamma. \tag{1}$$

THE INVERSE PROBLEM

In order to resolve the unknown location and shape of the interrogated scatterer, we seek to minimize, subject to the governing Helmholtz equation, the misfit functional defined either by:

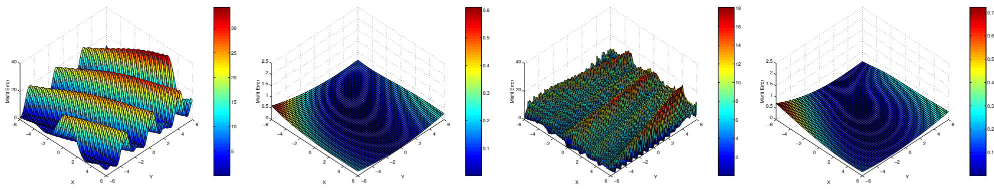
$$\mathcal{L}_1(\Gamma) = \frac{1}{2}\sum_{m=1}^{M} \frac{|u^s(\mathbf{x}_m) - u^s_m(\mathbf{x}_m)|^2}{|u^s_m(\mathbf{x}_m)|^2} \text{ or } \mathcal{L}_2(\Gamma) = \frac{1}{2}\sum_{m=1}^{M} \frac{||u^s(\mathbf{x}_m)| - |u^s_m(\mathbf{x}_m)||^2}{|u^s_m(\mathbf{x}_m)|^2} \tag{2}$$

In (2), $\mathbf{x}_m$ denotes the location of the stations, u^s_m denotes the measured scattered field at the same points, and u^s denotes the forward solution (computed), also at the M locations. Numerically, we experimented with both functionals (2) and opted to carry out the location and shape detection using $\mathcal{L}_2$ to avoid the highly oscillatory nature of $\mathcal{L}_1$ at high frequencies that is responsible for very narrow valleys of attraction and multiple minima (Fig. 1).

NUMERICS

Implementation

To seek a minimum to the functionals (2) we used a conjugate-gradient method (Polak-Ribiere variant); for the computation of the gradient functionals we used a finite difference scheme. For each update of the design parameters (location and shape parameters) the forward problem needs to be solved; we implemented the integral equations (1) using quadratic isoparametric boundary elements. Throughout all numerical experiments we chose 3 to 5 station locations along a fixed



(a) $ka = 1.0, \mathcal{L}_1$ (b) $ka = 1.0, \mathcal{L}_2$ (c) $ka = 5.0, \mathcal{L}_1$ (d) $ka = 5.0, \mathcal{L}_2$

Figure 1. Distribution of misfit functionals $\mathcal{L}_1$ and $\mathcal{L}_2$ (k: wave number; a: radius of the scatterer)

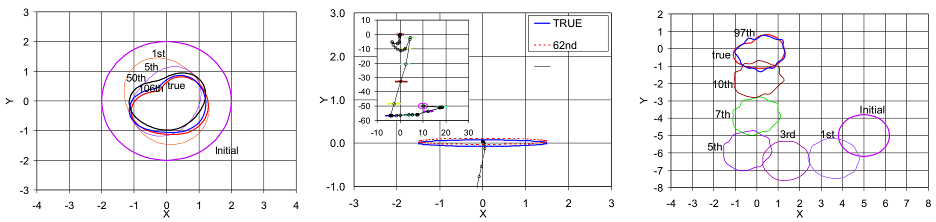
y-coordinate; the motivation stems from practical considerations, for it is rare that sensor distributions circumscribe a scatterer.

Frequency- and directionality-continuation scheme

Given a frequency and a set of initial guesses for the unknown shape and location parameters, our scheme for a single frequency is encapsulated in a standard algorithm. However, even if convergence is achieved, there is no guarantee that the converged parameters are the true ones. To improve, we iterate on the frequency (or wavenumber) space and use the converged parameters of the last frequency as initial guesses for the next frequency. Once the highest frequency of the frequency continuation scheme has resulted in converged parameters, we revisit the misfit for each one of the frequencies we considered earlier and re-compute the misfit for the final parameter values. If the computed values of the new misfits are less than the ones we obtained at the end of the previous convergence cycle, we consider the scheme to have converged. Optionally, the same scheme can be used for iterating over user-chosen directions of the interrogating waves, either as a stand-alone scheme, or in conjunction with the frequency continuation.

Numerical Results

We used the continuation schemes to resolve the location and shape of various scatterers and report here a subset of our results. In all cases of using $\mathcal{L}_2$, convergence to the true parameters was attained using a small number of frequencies. Though we have observed sensitivity of the number of iterations to the initial guess, we have not been able to construct a problem for which the continuation scheme would fail. Figure 2 depicts the detection scheme for various scatterers including a parametrically defined arbitrarily-shaped scatterer (Figure 2 (a)), a parametrically defined penny-shaped scatterer (Figure 2 (b)) and completely arbitrary geometries (Figure 2 (c)). For completely arbitrary geometries based on scatterer boundary discretizations, a non-self-intersecting shape algorithm was implemented to eliminate odd shape solutions.



(a) Arbitrarily-shaped scatterer (6 parameters) (b) Penny-shaped scatterer (4 parameters) (c) Arbitrarily-shaped scatterer (42 nodal coordinate parameters)

Figure 2. Numerical experiments for various scatterer shapes

CONCLUSIONS

We investigated numerically and reported on the algorithmic stability of frequency- and directionality-continuation schemes for detecting the location and shape of insonified scatterers using a small number of measurement stations. The results demonstrate that the schemes allow robust determination of the unknown shape using a small number of frequencies and directions of the probing insonifying waves.

References

[1] Colton D., Coyle J., Monk P.: Recent Developments in inverse scattering theory. *SIAM Review* **42**:369–414, 2000.
[2] Angell T.S., Jiang X., Kleinman R.E.: A Distributed Source Method for Inverse Acoustic Scattering. *Inverse Problems* **13**:531–545, 1997.
[3] Potthast R.: A Point-Source Method for Inverse Acoustic and Electromagnetic Obstacle Scattering Problems. *IMA Journal of Applied Mathematics* **61**:119–140, 1998.