THE EFFECT OF VISCOSITY ON THE PROPAGATION OF ACOUSTIC WAVES THROUGH FINE CYLINDRICAL MESHES

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Summary In this paper we discuss the effect that viscosity has on the propagation of acoustic waves through meshes consisting of small diameter cylindrical elements. The acoustic Reynolds number in such cases is extremely small except at very high amplitudes. Propagation which is parallel or perpendicular to the cylinder axis are both consider together with propagation through a mesh.

INTRODUCTION

Sound absorbing materials often involve fine scale structures in which the spacing between elements is so fine that the drag forces due to viscous effects are significant. In many cases the structures consist of nearly cylindrical elements arranged in a variety of different ways. In such cases the effect of the drag is twofold. First it reduces the propagation speed of the acoustic waves. Secondly it introduces an acoustic damping in which energy in the acoustic waves is converted into heat. The former can also help at low frequencies where smaller dimensions are required than is generally the case. In this paper we discuss the effect of the viscosity on the propagation characteristics of acoustic waves travelling through cylindrical meshes of arbitrary geometry. The paper is split into three parts: the effect of viscosity on the propagation of acoustic waves travelling parallel to the axes of an array of rigid cylinders; the effect of viscosity on the propagation of acoustic waves travelling perpendicular to the axes of a rigid cylinder; and the effect of propagation through a mesh consisting of rigid cylinders.

THE EFFECT OF VISCOSITY ON THE PROPAGATION OF ACOUSTIC WAVES TRAVELLING PARALLEL TO THE AXES OF AN ARRAY OF RIGID CYLINDERS

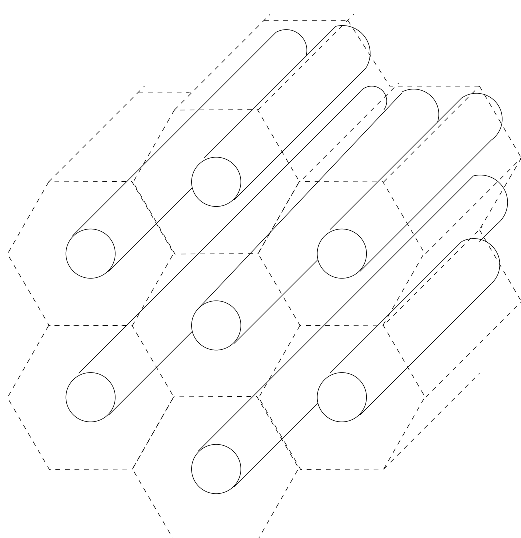

Figure 1. Propagation along an array of cylinders.

In the first part of the paper we consider the situation illustrated in figure 1. The spacing between the cylinders is sufficiently small that the acoustic boundary layer on each cylinder is significant and the amplitude of the acoustic waves combined with the cylinder size are such that the acoustic Reynolds number is low. We account for the attenuation of the sound field by viscous effects by solving the viscous Navier Stokes equation following Rayleigh [5] and others, notably Lighthill[4]. Here, however, the inner no slip boundary is a cylinder, rather than a plane surface and the outer boundary consists of set of periodic boundaries, representing the interaction of adjacent cylinders, and forms a regular polygon, as illustrated by the dashed lines in figure 1. The number of sides of the polygon is arbitrary in the analysis and depends upon the geometry. The momentum equation [2] results in a solution of the form:

$$u_z(r,\theta) = \sum_{\alpha=-\infty}^{\infty} \left(A_\alpha Ke_\alpha \left(\sqrt{\frac{\omega}{\nu}} r \right) + B_\alpha Be_\alpha \left(\sqrt{\frac{\omega}{\nu}} r \right) \right) \left(C_\alpha e^{i\alpha\theta} + D_\alpha e^{-i\alpha\theta} \right) - \left(\frac{1}{\rho i \omega} \right) \frac{\partial p}{\partial z} \qquad (1)$$

where u_z is the axial acoustic velocity, ω is the angular frequency, ν is the kinematic viscosity, r is the radius, p is the pressure, ρ is the density and θ is the angle. $A_\alpha, B_\alpha, C_\alpha$, D_α and α are unknown constants, and the functions Ke_α and Be_α are defined as:

$$Ke_\alpha(x) = ker_\alpha(x) + ikei_\alpha(x); \qquad Be_\alpha(x) = ber_\alpha(x) + ibei_\alpha(x) \tag{2}$$

where ber, bei, ker and kei are the Kelvin (or Thomson) functions [1, 3].
The unknown coefficients, A_α and B_α are found by applying the boundary conditions and using colocation. The effects of cylinder spacing, number of periodic boundaries, frequency and cylinder diameter are all discussed in terms of three non-dimensional parameters, $\sqrt{\frac{\omega}{\nu}}r_1$, $\frac{x_2}{r_1}$ and N (the number of sides). Here x_2 is the perpendicular distance to the periodic boundary.

THE EFFECT OF VISCOSITY ON THE PROPAGATION OF ACOUSTIC WAVES TRAVELLING PERPENDICULAR TO THE AXES OF A RIGID CYLINDER

In the second part of the paper the we consider the situation illustrated in figure 2. As before the acoustic Reynolds number is assumed low. The spacing is also considered to be sufficiently large for the cylinders to be independent of each other. The solution is obtained following the procedure used by Stokes [6] to calculate the steady drag on a sphere but including the unsteady term. It is worth noting that whilst a solution does not formally exist in Stokes' limit for a cylinder, one does exist for the unsteady problem described in the paper.

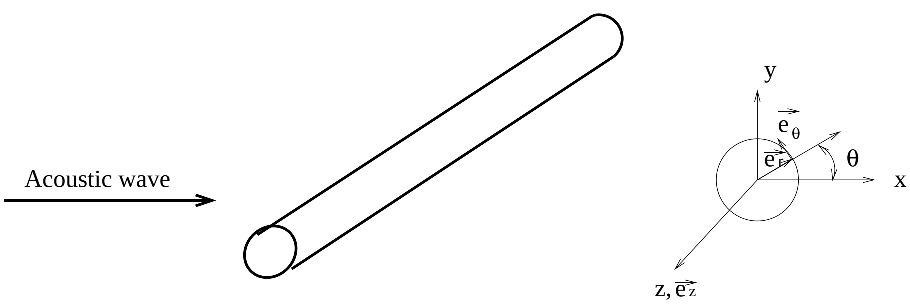

Figure 2. Propagation across an isolated cylinder.

PROPAGATION THROUGH A MESH CONSISTING OF RIGID CYLINDRICAL ELEMENTS

Finally, we combine the two analyses to obtain a model of the propagation through a mesh noting that both models are linear and so can be linearly summed.

CONCLUSIONS

The effect of viscosity on propagation of acoustic waves through fine cylindrical meshes is considered in detail. Two major effects are noted in the study. First the propagation speed of the acoustic waves is reduced by an order of magnitude, a helpful feature at low frequencies. Secondly considerable attenuation of the acoustic energy is observed. We also note that, although a secondary effect, the effect of polygonal periodic boundaries is noticeable. The number of boundaries, however, has little effect on the propagtion characteristics.

References

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