# STRUCTURE OF SONIC BOOMS IN A MEDIUM WITH MULTIPLE RELAXATION MODES

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*Summary* The inner shock structure of a sonic boom propagating through the atmosphere is controlled by dispersion and dissipation associated with the relaxation modes of oxygen and nitrogen. Using asymptotic and numeric methods, the shock structure is analysed and the change in shock thickness along the ray path is determined.

Accurate predictions of shock overpressure and shock rise-time are important in determining the subjective annoyance of sonic booms produced by supersonic aircraft. To assess the problems associated with sonic booms at ground level, the propagation of disturbances over long ranges must be investigated for a realistic atmosphere. In particular the effect of atmospheric parameters on the shock amplitude and shock thickness/rise-time must be predicted accurately.

For a supersonic body, a disturbance is generated in the form of a Mach cone. Close to the body, the exact form of the disturbance depends on the detailed geometry of the body, but over a number of wavelengths, the effect of the quadratic nonlinearity is to form a N-wave. Predictions of shock overpressure and shock rise-time are important in determining the subjective annoyance of sonic booms at the ground. Hence, the propagation of disturbances over long ranges must be investigated for a realistic atmosphere. In particular the effect of atmospheric parameters on the shock amplitude and shock thickness/rise-time must be predicted accurately. The evolution of the disturbance along ray paths normal to the shock front is governed by the combined effects of geometrical spreading, finite-amplitude effects, stratification and diffusion/dispersion due to molecular processes. For acoustic propagation through the atmosphere, thermoviscous diffusion does not fully describe the complex molecular processes present and relaxation due to internal vibration energies of polyatomic molecules must be included. Each relaxation mode is characterised by a relaxation time $\tau$ and the difference $\Delta$ between low- and high-frequency linear sound speeds.

Ignoring the effect of stratification, the governing equation for the perturbation pressure, suitably non-dimensionalised, is

$$\frac{\partial p}{\partial T} - p\frac{\partial p}{\partial X} + j\frac{p}{2T} = \delta\frac{\partial^2 p}{\partial X^2} + \sum_{\nu} \Delta_\nu e^{-X/\tau_\nu} \int^X e^{Y/\tau_\nu} p_{YY}(Y,T)\, dY, \qquad j = \begin{cases} 0 & \text{Plane Wave,} \\ 1 & \text{Cylindrical Wave.} \end{cases}$$

where $\delta$ is the coefficient of thermoviscous diffusivity and the summation is over the number of relaxation modes present. For a single relaxation mode, ignoring viscosity, it is well known that the shock structure can be classified as either partly- or fully-dispersed depending on the shock amplitude and the relaxation parameters [1, 2]. For fully-dispersed shocks, the relaxation mode determines the entire shock structure, while in the case of partly-dispersed shocks relaxation alone is insufficient to support the whole shock and a viscous sub-shock arises. Thus transition from a full-dispersed to a partly-dispersed shock corresponds to a sharp decrease in shock thickness.

For the case of a single relaxation mode, taking into account the effect of stratification, it has been shown that the shock rise-time is very sensitive to the relaxation parameters [3]. Indeed, along certain ray paths the shock structure can change from partly-dispersed, to fully-dispersed, then finally becoming partly-dispersed again, with a resulting change in shock thickness. However, for air, relaxation modes associated with oxygen and nitrogen are both significant. Thus in the present paper we consider the shock structure for two relaxation modes using both asymptotic and numeric methods.

For parameter values typical of sonic booms in the atmosphere, we find $\tau_1 \gg \tau_2 \gg \delta$, where $\tau_1$ is the non-dimensional relaxation time associated with $N_2$ molecules and $\tau_2$ the relaxation time for $O_2$ molecules. Thus an asymptotic analysis is possible. First travelling wave solutions are obtained. The structure of the solution is found to depend on the relative

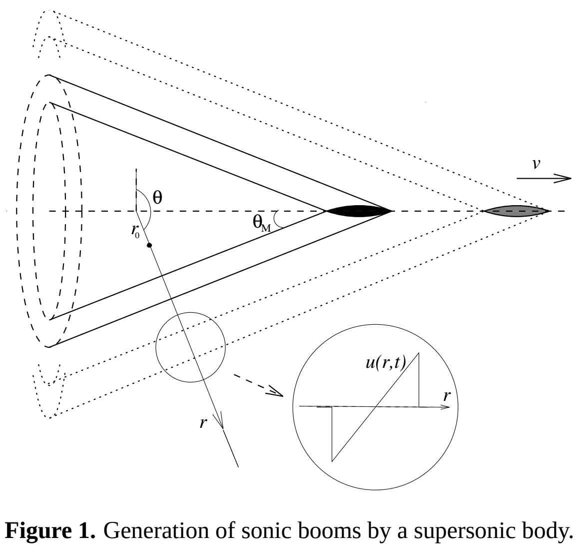


**Figure 1.** Generation of sonic booms by a supersonic body.

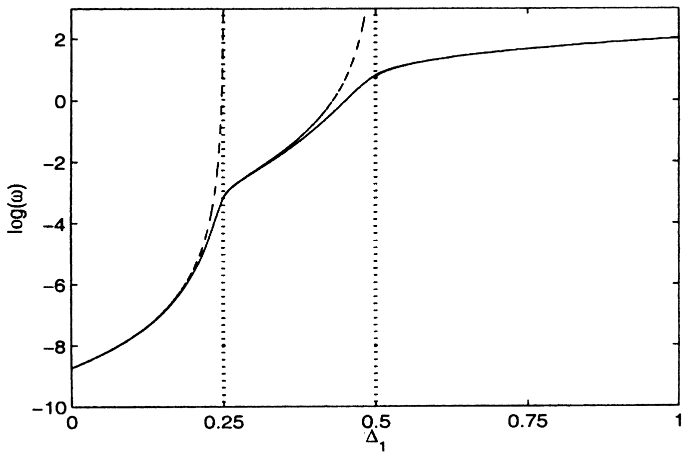


**Figure 2.** Plot of shock width as a function of $\Delta_1$ for travelling wave solution with $\Delta_2 = 0.25$, $\tau_1 = 1$, $\tau = 0.01$ and $\delta = 5 \times 10^{-6}$.

sizes of $\Delta_1$ and $\Delta_2$. Considering first a plane wave, for $\Delta_1 > 1/2$, the shock is described entirely by the fully-dispersed solution associated with the $N_2$ relaxation mode. For $\Delta_1 < 1/2$ and $\Delta_1 + \Delta_2 > 1/2$, the outer shock is a partly-dispersed $N_2$ solution with an embedded sub-shock consisting of a fully-dispersed $O_2$ solution. Finally for $\Delta_1 < 1/2$ and $\Delta_1 + \Delta_2 < 1/2$, the shock structure is particularly intricate with an outer partly-dispersed $N_2$ solution, which contains a narrower sub-shock controlled by the $O_2$ relaxation mode and embedded in this is a Taylor-like viscous shock. This behaviour is illustrated in figure 2, where the shock width (defined as the minimum value of $\frac{\partial p}{\partial X}$) is plotted against $\Delta_1$ for fixed $\Delta_2$. The solid line shows the numerical result obtained by shooting.
Including geometric effects and solving the PDE from an initial N-wave signal, asymptotic solutions are determined, with the shock structure depending now on shock amplitude (which is dependent on the history of propagation) as well as on the local parameters $\Delta_1$ and $\Delta_2$. Numeric results were obtained using finite-difference schemes. However, for the small values of $\tau_1, \tau_2$ and $\delta$ considered, computation was extremely time consuming due to the fine length scales within the shock. Hence a variable mesh scheme was used, utilising knowledge of the shock position from the aymptotic results. Figure 3 shows the change in shock width with propagation distance. The solid line is the numerical result. Asymptotic theory predicts that for $T < 2.78$ the shock structure consists of a narrow $O_2$ sub-shock within a partly-dispersed $N_2$ shock, but that for $T > 2.78$ the $N_2$ shock becomes fully-dispersed and hence the shock loses its finer scale.

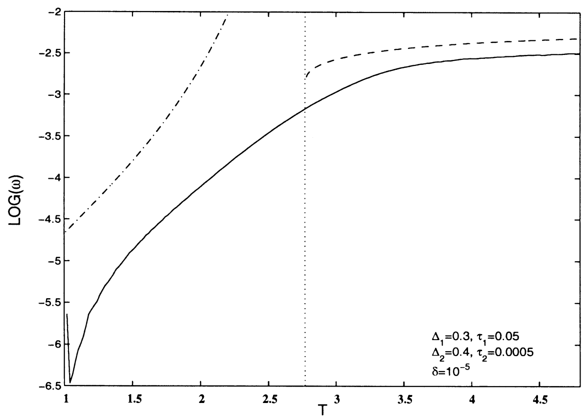


**Figure 3.** Plot of shock width as a function of $T$ for $\Delta_1 = 0.3$, $\Delta_2 = 0.4$, $\tau_1 = 5 \times 10^{-2}$, $\tau_2 = 5 \times 10^{-4}$ and $\delta = 10^{-5}$.

Further results presented show that for certain parameter ranges, a combination of asymptotic and numeric methods is the only practical approach for accurate prediction of shock rise-time. Numerics alone prove impractical due to the need to resolve three different lengthscales within the shock, while in some cases the asymptotic analysis must be supplemented by numeric solutions of the disturbance outside the shock region.

## References


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