SHOCK LEAKAGE THROUGH A VORTEX-LADEN MIXING LAYER CAUSING JET SCREECH

Takao Suzuki*, Sanjiva K. Lele**

*California Institute of Technology, Engineering and Applied Science, Pasadena, CA 91125, USA

**Stanford University, Aeronautics and Astronautics, Stanford, CA 94305-4035, USA

Summary The interaction between shock-cells and vortices in a mixing layer of a jet generates intense shock noise causing jet screech. This paper studies this shock noise phenomenon using a geometrical theory for weak shock motion. The unsteady ray-tracing equation shows that the local vorticity in the mixing layer behaves as a barrier and the standing shock leaks near the saddle points between vortices. Direct numerical simulation demonstrates this by showing good agreement of the shock front with the geometrical theory.

INTRODUCTION

When a supersonic jet is under-expanded, a shock-cell structure generated due to pressure mismatch interacts with the vortices convected in a mixing layer of the jet. As a result, some acoustic energy associated with the shock leaks through the mixing layer and, in turn, radiates as intense noise in the far field. When the radiated sound propagating upstream excites another instability wave from the nozzle lip, the whole system creates a feedback loop, known as "jet screech." The frequency of the screech tone has been successfully predicted in past [1], but the mechanisms of intense sound generation remain somewhat elusive.

Although several previous studies have modeled the shock noise as non-linear interaction between the shock-cell structure and shear layer instability waves, those models was not able to explain the intense noise generation nor provide quantitative prediction. A recent work [2] introduced geometrical acoustics and computationally investigated the sound radiation pattern of shock noise with the linearized Euler equations. However, the validity of the high frequency limit for standing shock waves was uncertain, and the shock noise amplitude was still unpredictable.

The objective of this study is to identify the mechanism of intense shock noise generation and to predict the amplitude as well as the wave-front evolution. The key idea here is to demonstrate that the governing equations for weak shock motion are identical to the equations derived from geometrical acoustics. We validate the model based on the geometrical theory by performing direct numerical simulation (DNS) in two-dimensions (refer to [3] for details).

MODEL OF SHOCK LEAKAGE THROUGH A VORTEX-LADEN MIXING LAYER

When the angle of waves (defined based on the wave-front normal) issued from the potential core is higher than $\theta^{cr} \equiv \cos^{-1}[-a_{\rm jet}/(a_\infty + u_{\rm jet})]$, total internal reflection occurs in a supersonic jet. Since the Mach angle, $\theta^M \equiv \cos^{-1}(-1/M_{\rm jet})$, is always higher than this angle, the shock-cell structure should be confined in the potential core. However, as instability waves change the vortical structure of the mixing layer, a portion of shock can "leak" across the mixing layer.

To describe the shock motion, we introduce a geometrical theory, which is analogous to the high frequency limit of acoustics. We solve the evolution of a pressure jump expressed by a geometrical form (i.e. $p(t,\mathbf{x}) = P(t,\mathbf{x})h[\phi(t,\mathbf{x})/\epsilon]$, ϵ being the width of the shock and $h[\phi/\epsilon]$ a ramp function). Substituting it into the Euler equations, the leading order terms of $\epsilon \to 0$ become the eikonal equation (the ray-tracing equation) for unsteady flows:

$$(\phi,_t + u_j \phi,_j)^2 - a^2 \phi,_j^2 = 0 \, . \tag{1}$$

In turn, the equation for constant ϕ solves a line of discontinuity corresponding to the shock-front. Subsequently, the second highest order terms yield the first order transport equation, in which the quantity

$$\frac{P^2 S}{p(\phi,_t + u_k \phi,_k)} \sqrt{a_\infty^2 + |\frac{d\mathbf{x}}{dt}|^2} \tag{2}$$

is conserved along ray tubes in space-time coordinates (where S denotes the area of the tube in a four-dimensional sense). This quantity corresponds to the "Blokhintzev invariant" [4] in four dimensions, which provides the intensity of the shock. Thus, we can solve the unsteady shock motion as well as its intensity by the method of characteristics.

By solving the equation of the wave-front normal, we find that the local vorticity rotates the shock-front: If the vorticity distributed along the shock trajectory is high, the trajectory creates a loop and total reflection occurs. In contrast, rays can leak near the saddle points of vortices where the local vorticity is thinner. As the vortices are convected, the reflection site of the shock-cell structure is pinched off periodically and this portion is radiated as intense tonal noise.

NUMERICAL METHODS

We demonstrate the geometrical theory by performing DNS of an iso-thermal supersonic mixing layer ($M_{\rm jet} = 1.2$) in two-dimensions. In one case we directly solve the shock motion by imposing a standing shock in DNS, while in the other case we calculate the unsteady ray tracing equation and apply the ray tube theory (i.e. the Blokhintzev invariant) with the DNS background flow data. These two cases are compared in terms of the shock-front evolution and shock amplitude.

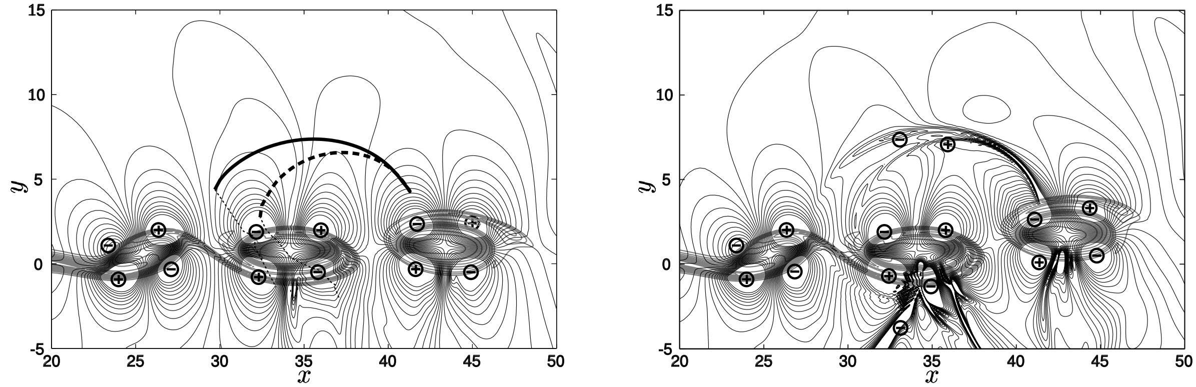

Figure 1. Comparison of the shock noise from a mixing layer. Shock-fronts calculated with the eikonal equation on the left, and that from DNS on the right. Vorticity contours are superposed on dilatation field: $(+)$, expansion; $(-)$, compression.

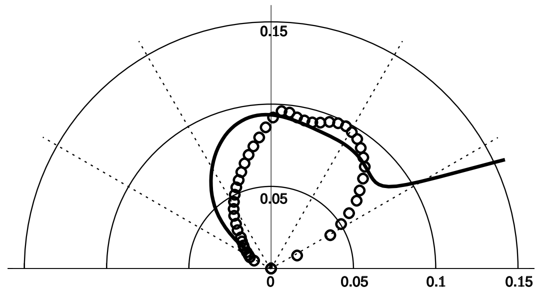

Figure 2. Comparison of shock wave amplitude. Pressure jump of the shock at distance $R \approx 14\delta_V$ centered at $(35.0, 0)$ was measured, and the pressure ratio defined as $(\Delta p_{R=14})/(\Delta p_{y_{\text{initial}}})$ is plotted: ——, geometrical theory; $\circ$, DNS. $\Delta p_{y_{\text{initial}}} \approx 0.1 p_\infty$

RESULTS OF SIMULATION

Figure 1 compares the shock-fronts obtained from the eikonal equation and from DNS and shows good agreement in which the standing shock wave starts to leak near the saddle point and forms a compression branch followed by an expansion branch. This circular wave propagates in the far field with nearly the speed of sound. Since the geometrical theory assumes linearity, the wave-fronts obtained from the eikonal equation tends to delay compared with those calculated with DNS.

The comparison of shock amplitude in figure 2 shows that the overall sound pressure level of DNS is comparable to that of the geometrical theory, although the detailed directivity pattern has slight discrepancy. Since a portion of the shock itself changes into sound wave, its intensity becomes much higher than that generated by the non-linear flow interaction. We can also observe that appreciable amplitude of shock noise propagates upstream, which potentially causes the feedback mechanism of jet screech.

CONCLUSIONS

This study has theoretically and computationally demonstrated that standing shock waves in a potential core can most easily leak near the saddle points of vortices in a supersonic mixing layer. Previous experimental studies have indicated that the shock noise is most intensive somewhere between the second and fourth shock-cell locations. We can interpret this trend from the current study: Shock waves can leak through the mixing layer only after the unsteady vortices have sufficiently developed. However, when smaller scale eddies develop further downstream, the organized vortical structure is disrupted so that distinctive saddle points are no longer formed, and the shock noise intensity is reduced again. Instead, broadband shock noise is generated due to scattering by turbulence.

As mentioned, the difference between the Mach angle and the critical angle of total reflection is given by a function of the jet Mach number and the temperature ratio of the mixing layer. This relation indicates that the threshold for shock leakage becomes lower as the jet temperature increases, which is consistent with the previous observation that screech noise becomes intensive downstream for heated jets.

References

[1] Powell A.: On the mechanism of choked jet noise. *Proc. Phys. Soc. Lond.* B **66**:1039–1056, 1953.
[2] Manning T.A., Lele S.K.: A numerical investigation of sound generation in supersonic jet screech. *AIAA paper*: AIAA-2000-2081, 2000.
[3] Suzuki T., Lele S.K.: Shock leakage through an unsteady vortex-laden mixing layer: application to jet screech. *J. Fluid Mech* **490**:139–167, 2003.
[4] Blokhintzev B. I.: The propagation of sound in an inhomogeneous and moving medium. *J. Acoust. Soc. Am.* **18**:322–328, 1946.