WAVE PROPAGATION IN AND SOUND EMISSION FROM A SANDWICH PLATE UNDER HEAVY FLUID LOADING

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Summary The talk addresses wave motion in an unbounded fluid-loaded elastic sandwich plate composed by two identical isotropic skin plies and an isotropic core ply in a general three-dimensional problem formulation. Several alternative theories for stationary dynamics of such a plate are suggested, including a formulation in the framework of a theory of elasticity applied for the core ply. In the first instance, a fluid loading at the both sides of a plate is considered and 'in-phase' and 'anti-phase' wave motions (with respect to transverse deflections of skins) are analysed independently upon each other. Dispersion curves are compared and it is shown that the simplified models are capable to give a complete and accurate description of all propagating waves in not too high frequency range, which is sufficient in practical engineering. The role of sound emission into the acoustic medium in wave propagation phenomena is studied. Furthermore the analysis is extended to take into account for the 'symmetry-breaking' effects, e.g., a static pre-stress of one of skin plies and a fluid loading at one side of the sandwich plate. The standard perturbation technique is applied to analyse an interaction between dispersion curves in these cases.

INTRODUCTION. THE PROBLEM FORMULATION

Sandwich plates are widely used in many technical applications because this composition of a thin-walled structure conveniently combines the properties of a high strength and a low weight. The issues of strength and reliability of such structures have been thoroughly studied in many publications. The stability and dynamics of sandwich beams and plates have also been explored quite extensively. However, the effects of heavy fluid loading (which are particularly relevant to various naval applications) have not yet been studied in details. There are two possibilities to describe dynamics of sandwich plates. Equations of motions of a core ply may be formulated in the framework of a theory of elasticity, whereas in-plane and out-of-plane dynamics of each skin is governed by reduced equations of elasto-dynamics and by a conventional plate theory, respectively. This system of equations of the 'refined' theory is supplemented by continuity conditions at the interfaces between plies. Alternatively, some hypotheses may be adopted concerning the deformation of an arbitrary cross-section of the whole package of plies and reduced low-order equations are then derived in a 'simplified' theory. Inasmuch a heavy fluid loading is included, additional continuity conditions are formulated at fluid-structure interfaces and the wave equation is introduced for an acoustic medium in both the 'refined' and the 'simplified' theories. Naturally, the 'simplified' formulation does not permit to capture short-wave high-frequency motions in sandwich plates, but the essential (from the practical viewpoint) features of wave propagation in these structures are related to the audio frequency range. In this range, the length of a propagating wave exceeds a thickness of the whole package of plies at least in 3-4 times. Thus, for low and intermediate frequencies, it is expedient to estimate the actual validity of the 'simplified' theories in describing wave motions in a sandwich plate under heavy fluid loading. Three small non-dimensional parameters are introduced to characterise the internal structure of a sandwich plate: the geometry parameter ε as a ratio of the thickness of a skin to the thickness of a core, the density parameter δ as a ratio of the density of a core to the density of a skin and the stiffness parameter γ as a ratio of the Young's module of a core to the Young's module of a skin. Each of these parameters is small and it facilitates an asymptotic analysis of dispersion equations of the 'refined' theories, which proves that the 'simplified' theories in effect deliver the same dispersion equations up to the leading order terms. Stationary wave propagation is considered and the frequency parameter is defined as $\Omega = \dfrac{\omega h_{skin}}{c_{skin}}$ (c_{skin} is a sound speed in the material of skin plies).

THE SYMMETRIC CASE

In the absence of static pre-stress of skins, two uncoupled classes of waves exist in a plate of symmetric composition, which is fluid-loaded at both sides. They may conveniently be characterised by a phase shift between the lateral displacements of skin plies. The 'in-phase' waves are specified as those which preserve the thickness of a sandwich plate (they characterise 'global' flexural motions of the whole package of plies), whereas the 'anti-phase' waves are developed if the lateral displacements of skins are in opposite direction to each other at any instant of time.

'In-phase' waves
A dominantly flexural propagating wave, which exists in a fluid-loaded sandwich plate at an arbitrary low frequency, is adequately captured by the 'simplified' theory. Besides, two propagating waves of dominantly shear deformation

emerge in the 'low-frequency range' ($\Omega < 0.5$). Their cut-on frequencies are controlled by the parameters ε, γ of the sandwich plate composition and by the compressibility parameter defined as a ratio of the sound speed in an acoustic medium to the sound speed in the material of skins. The magnitudes of these cut-on frequencies and location of dispersion curves are also adequately predicted by the 'simplified' theory when $\gamma < 0.005$, which is a typical situation for the sandwich plates used in technical applications.

'Anti-phase' waves

In this case, the methodology to derive the 'refined' theory is not altered as compared with the previous case, whereas three 'simplified' theories are derived independently upon each other to describe the dominantly dilatational, the dominantly shear and the dominantly flexural wave. The dependence of their cut-on frequencies upon the parameters of sandwich plate composition and on the compressibility parameter is studied. A good agreement between the results obtained by a use of the 'refined' and the simplified' theories is observed in the whole 'low-frequency range' ($\Omega < 0.5$) except for relatively narrow frequency bands, where the dispersion curves predicted by 'simplified' theories cross each other. In these zones, more complicated behaviour of dispersion curves (including generation of the waves with phase and group velocities in opposite directions) detected by a use of the 'refined' theory, explains the failure of the 'simplified' ones.

TWO 'SYMMETRY-BREAKING' CASES

The results reported in the previous section show that several elementary theories, each of which is derived from *a-priori* hypotheses of kinematics of plate's deformation are capable to provide a complete and accurate description of all propagating waves in not too high frequency range, which is sufficient in practical naval and aerospace engineering. In practice, it is often the case that only one skin is pre-stressed or, (e.g., for a ship's hull) a compressible fluid is located only at one side of a sandwich plate. Then the 'in-phase' and 'anti-phase' wave are not decoupled from each other, but it is possible to apply standard perturbation technique to study an interaction between different branches of the dispersion curves. An attention is focused at the analysis of the modes in the intersection cases (when the individually taken 'in-phase' and 'anti-phase' dispersion curves cross each other). It is shown that it does not present serious difficulties to predict location of all dispersion curves in the low-frequency range for the both considered cases by a use of perturbation methods.

THE ROLE OF EMISSION OF ACOUSTIC WAVES

The comparison of dispersion curves obtained for various values of the sound speed ratio sheds a light on the role of emission of acoustic waves. As is found out this phenomenon influences propagation of the dominantly in-plane waves (both 'in-phase' and 'anti-phase' ones) stronger than the propagation of the dominantly flexural waves. In particular, the threshold sound speed ratio is found, which (at the fixed excitation frequency) separates the regime of propagation of a trapped wave and the regime of its decay. In the case of an incompressible fluid, there is no energy flow from a plate to infinity in the direction perpendicular to the surface of a vibrating plate. As soon as the fluid becomes sufficiently compressible, the energy transmission ('leakage') from the plate into surrounding acoustic medium is developed. Respectively, the structural energy flow disappears and the free propagating wave cannot be supported in plate under such a fluid loading.

CONCLUSIONS

The set of 'simplified' theories is suggested to describe wave propagation phenomena in sandwich plates under heavy fluid loading. Comparison of dispersion curves obtained by solving characteristic equations derived from these theories with dispersion curves obtained in the 'refined' problem formulation shows that these theories predict adequately wave motions in a sandwich plate in the whole frequency range of practical interest. It is shown that dominantly in-plane waves may either be propagating or decaying depending on the magnitude of the compressibility parameter (the ratio of a sound speed in a fluid to a sound speed in skin's material) and a simple explanation of this transition is given.

References

[1] Sorokin S. V.: A Note on the Effect of Compressibility on the Propagation of Free Waves in Sandwich Plates with Heavy Fluid Loading *J. Sound and Vibration*: corrected proof available on-line.
[2] Sorokin S. V.: Analysis of Wave Propagation in Sandwich Plates with and without Fluid Loading. *J. Sound and Vibration*: corrected proof available on-line.