NOCTURNAL TEMPERATURE INVERSIONS UNDER CALM CLEAR CONDITIONS: AN ANALYTICAL STUDY

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Summary The knowledge of inversion height in the nocturnal boundary layer (NBL) under calm clear conditions is crucial in determining the fate of chemical pollutants that are (accidentally or otherwise) released into the atmosphere. A new analytical expression for temperature profiles over bare soil surfaces under calm clear conditions is used to study inversion height and intensity as a function of surface parameters like ground emissivity and cooling rates. Previous analytical expressions available in the literature have ignored these parameters.

INTRODUCTION

On clear days the daily cycle of sun causes a diurnal variation of heating and cooling of the earth's surface which effect the immediate air layers above it creating a boundary layer. While the daytime convective boundary layer has been reasonably well studied (Garrat 1992) the same does not seem to be true for the nocturnal boundary layer (Ha and Mahrt, 2003). One of the main reasons for this is the complex nature of the radiative flux divergence. During the daytime the ground gets hotter quickly than the air because very less solar radiation is absorbed by air. On the other hand during nights the ground cools rapidly by radiating to the cooler space. This rapid cooling of the ground causes cooling of air above it by molecular conduction (because of near zero wind conditions). This results in a temperature profile that increases with height upto some level (say) z_{inv}. Therafter the temperature decreases with the usual adiabatic lapse rate provided all the daytime turbulent convection is eroded. This height z_{inv} is what we refer as the nocturnal inversion (Figure 1). This inversion acts as a lid that stops vertical motion and mixing. Consequently pollutants released within an inversion layer often travel long distances without much mixing and spreading causing severe disasters. One such disaster occured in Bhopal (India) during the early hours of 3, December 1984 when methyl isocyanate (MIC) escaped through a nozzle of 33 m high atmospheric vent-line from the Union Carbide plant. The nocturnal inversion at that time was estimated to be around 250m (Maithili Sharan et al., 1995) which is well above the height at which the lethal MIC got released. This affected about 250,000 people causing death to a significant proportion. Thus the importance of studying nocturnal inversion need not be over emphasized.

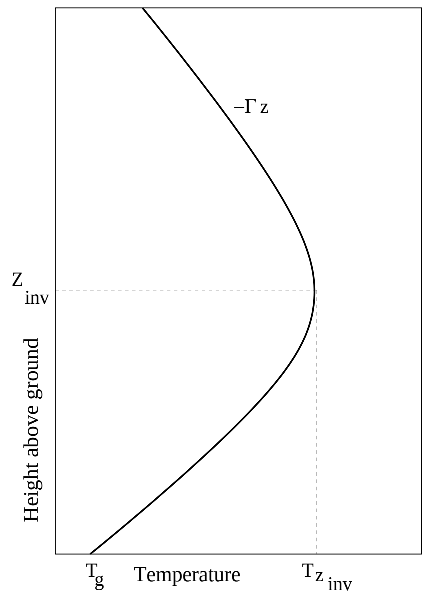

Figure 1. Schematic of typical nocturnal temperature distribution with radiation inversion.

THE ANALYTICAL SOLUTION

Assuming calm clear conditions, so that the nocturnal temperature profile is a function of the vertical height and time alone, Narasimha and Vasudeva Murthy (2003, hereafter NV) derived an approximate analytical solution using singular perturbation techniqiues. If $T(z,t)$ denotes the temperature at height z and time t the analytical expression is given by

$$T(z,t) = T_{g_0}[1+\Theta] - \Gamma z. \tag{1}$$

Here T_{g_0} is the initial ground temperature (for example, it could be just after sunset). The expression for Θ is

$$\begin{aligned} \Theta &= -\lambda - \epsilon_g\beta\sqrt{t} + \Gamma z + [\lambda - \Gamma z]e^{-\tau e_m(z)} + [\lambda + (\epsilon_g - 1)\beta\sqrt{t}]e^{-\zeta} \\ &\quad -\lambda e^{-\tau} erfc\left(\frac{\zeta}{2\sqrt{\tau}}\right) - \frac{\lambda}{2}\left[e^{-\zeta} erfc\left\{\frac{2\tau-\zeta}{2\sqrt{\tau}}\right\} - e^{\zeta} erfc\left\{\frac{2\tau+\zeta}{2\sqrt{\tau}}\right\}\right] \end{aligned} \tag{2}$$

Here,

$$\lambda = (1-\epsilon_g)(1-\epsilon_s)/4, \tag{3}$$

ϵ_g is the ground emissivty and ϵ_s is the sky emissivity, β is the ground cooling rate and Γ is the adiabatic lapse rate. The function $e_m(z)$ is given by

$$e_m(z) = \frac{\exp(-\delta z)}{1+\alpha z} \tag{4}$$

where δ^{-1} corresponds to the scale height of water vapour mixing ratio and α^{-1} corresponds to the so called emissivity layer or the radiative boundary layer height. Finally ζ and τ are the boundary layer variables

$$\zeta = \epsilon^{-1/2} z, \quad \tau = \epsilon^{-1} t, \tag{5}$$

ϵ is the ratio of radiative to diffusive time scales and is the small parameter in the formulation, typical values are 10 s and 10^5 s (see NV). The smallness of ϵ indicates the dominance of radiative flux divergence in the nocturnal boundary layer.

Several analytical expressions have been proposed by various authors (see Ragothaman et al., 2002, hereafter R02) but none of them take into account surface parameters like $\epsilon_g, \epsilon_s, \alpha$ and β. R02 have also considered these parameters but it was based on numerical simulation. This simple analytical expression explicitly shows how the near surface temperature evolution depends on surface parameters.

RESULTS AND DISCUSSION

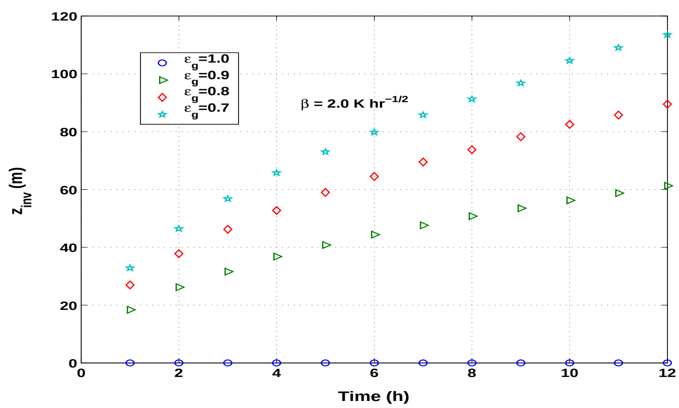

Figure 2. Effect of surface emissivity on the evolution of the inversion depth.

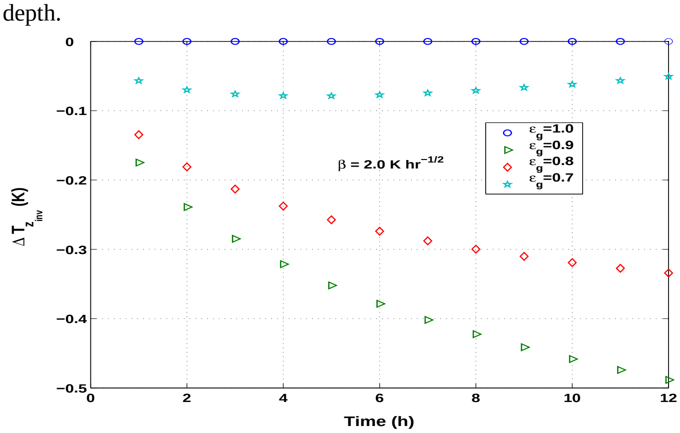

Figure 3. Effect of surface emissivity on the evolution of the temperature difference $\Delta T_{z_{inv}}$.

The above expression is used to compute the inversion height z_{inv} and the intensity of the inversion

$$\Delta T_{z_{inv}}(t) = T_{z_{inv}}(t) - T_g(t) \tag{6}$$

where T_g and $T_{z_{inv}}$ are the ground temperature and the temperature at z_{inv}. Two typical results are given in figure 2 and 3 which show the evolution of the above mentioned quantities for various emissivities of the ground. Except for $\epsilon_g \equiv 1$ the inversion height grows steadily and from the analytical expression it is clear that it grows like $\sqrt{t}$ whereas $\Delta T_{z_{inv}}$ decreases with increase in ϵ_g except at $\epsilon_g \equiv 1$. These results are qualitatively very similar to the results of R02 (for $\epsilon_g < 1$) which was based on numerical simulation of a more complex partial differential equation that included nonlinear integral terms. In deriving (1) NV have simplified the nonlocal nonlinear terms to local linear terms which was then subjected to singular perturbation analysis that yielded an approximate analytical solution. In R02 even for $\epsilon_g \equiv 1$ the inversion height showed a steady growth. Also $\Delta T_{z_{inv}}$ was always positive in R02 which means that the ground temperature is always less than the temperature at inversion height. These two lacunae could be probably due to the above mentioned simplification used in NV.

The importance of ϵ_g and β in the NBL has been recently well studied by Narasimha and his co-workers, see for e.g. Varghese et al. (2003). This was also hinted earlier by Elliot and Stevens (1966).

Acknowledgement

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