Multiple Bubbles Dynamics using Level Set Indirect Boundary Element Method

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Summary We present a new method for simulating bubble dynamics called Level Set Indirect Boundary Element Method (LSBEM). This method combines the advantages of LSM with BEM. As we know, the Level Set Method (LSM), which is a well known computational technique used for tracking a propagating interface over time, has the strength in accurately handling topological complexities and changes. The Boundary Element Method (BEM), on the other hand, is known to serve well in conserving computational effort by reducing the dimensions of the problem by one. The essence of this work is that while keeping this advantage of BEM, LSBEM simplifies the representation of the interface of multi-bubbles by using LSM. Thus advantages from both methods are utilised. A number of techniques are applied to ensure solution convergence and numerical accuracy. For instance, effort is made to avoid singularities in calculation by defining two sets of source and control points on the mesh which will never overlap one another; also work is done to ensure solution accuracy by reinitializing the level set function using Fast Marching Method (FMM) after every two time-steps.

INTRODUCTION

Boundary Element Method (BEM) is commonly applied in the simulation of bubble(s) dynamics as seen in the works of Blake & Gibson [1], Oguz & Prosperetti [2], and Zhang et al [3]. Most of these works were based on the so-called Direct Boundary Element Method (DBEM) in which the potential or its normal derivative is the primary unknown to be solved first. The method employed here, known as Indirect Boundary Element Method (IBEM), takes the source or dipole distributions as the primary unknown to be solved first, and then evaluate the potential and material velocity in the flow field. Another component to LSBEM is the Level Set Method which was introduced by Osher and Sethian in 1988 [4]. LSM traces the deformation of the interface, in this case the bubble surface, by advecting the level set function. This function is a scale function defined in the space with one dimension higher then that of the interface and the interface coincides with the zero level set. Therefore the LSBEM tries to combine both methods while retaining their respective strengths.

COMPUTATIONAL METHOD DEVELOPMENT

A background uniform mesh covering all the computational domain of the given problem is constructed. Then the level set function is defined on it with source points and control points along the interface (the zero level set). The source points are defined as grid points that are located inside the bubble with at least one neighbouring point that is not inside the bubble. The control points, however, are defined as the middle points of the segments whose two nodes are located in the different sides of the interface. In IBEM, the potential distribution is approximated as the sum of source strength at every source points, which is in turn decided by enforcing the potential value at each control point to be equal to the given boundary condition of the potential. Since the bubble is a close object, the total number of control points will always be larger than that of source points. Therefore, no exact solution can be found; but a least square type of solution can always be obtained.

After getting the source strength on each source point, the velocity distribution of the computational domain is generated. However, it must be noted that the velocity distribution inside the bubble is still unknown and thus has to be calculated by extending the velocity field into the bubble via interpolation. Once this is done, the level set function is shifted to the next time step for the new interface location. Based on the location, source and control points are updated by evaluating the Bernoulli's equation. In order to maintain the accuracy of the results, the level set function is reinitialized after two time-steps with the use of the Fast Marching Method (FMM) [5].

PRELIMINARY RESULTS

We have obtained results for one, two and three bubbles simulations. In this summary, we will show the results from three bubbles simulations. Fig.1 shows the three bubbles in an infinite medium without gravity at three different time-steps.

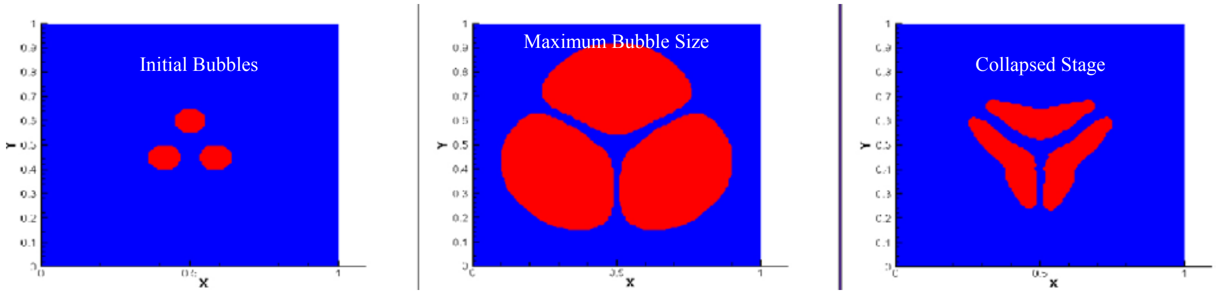

Figure 1: Simulation of three bubbles

Then we proceed to simulate three bubbles with the inclusion of a free surface (Fig.2) and also the simulation with gravity taken into consideration (Fig. 3).

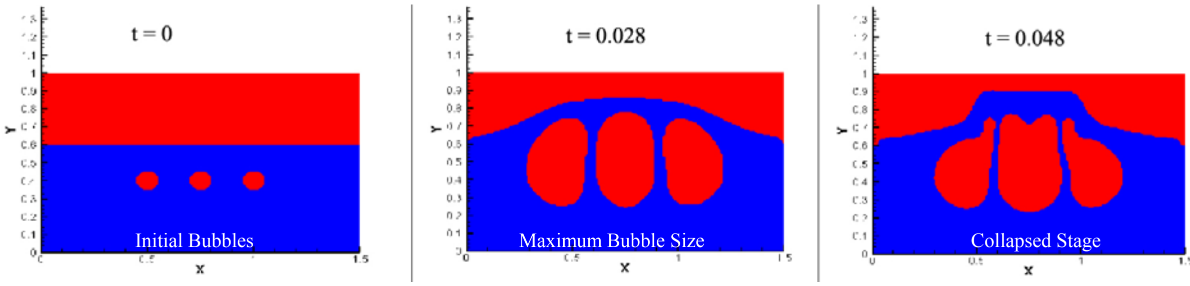

Figure 2: Simulation of three bubbles with a free surface on top

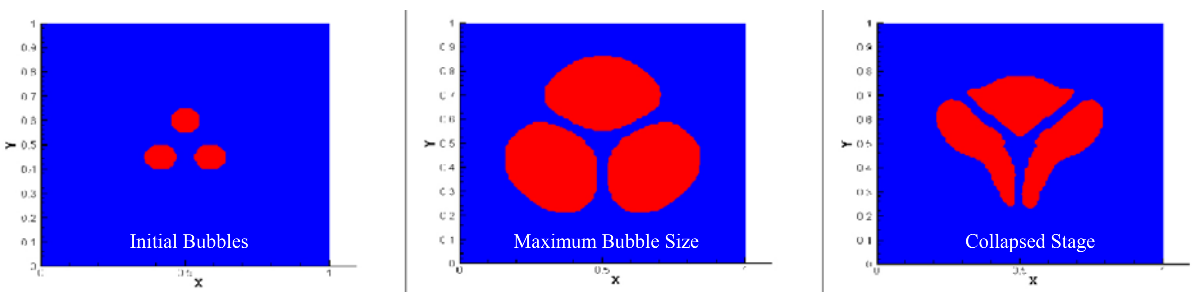

Figure 3: Simulation of three bubbles with gravitational consideration

CONCLUSIONS

Since the size of the matrix involved is determined by the number of source points generated, it is therefore still much less than the total number of grid points of the background mesh. This implies that the advantage of BEM is still kept in the LSBEM method. The representation of the interface by the level set functions simplifies the representation of the interface of multi-bubbles; again utilizing the strength of LSM. It is believed that LSBEM is ideal for simulating bubble dynamics involving topological changes in the multiple bubbles.

References

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