MODELLING SURFACE TENSION USING A GHOST FLUID TECHNIQUE WITHIN A VOLUME OF FLUID FORMULATION

Marianne M. Francois, Douglas B. Kothe, Sharen J. Cummins
Computer and Computational Sciences Division, Los Alamos National Laboratory, Los Alamos, NM 87545, U.S.A.

Summary We investigate a ghost fluid method in a volume of fluid formulation to model the surface tension force along the normal direction at an interface between two fluids of different properties. We compare it with the Continuum Surface Tension Force (CSF) method using numerical results to support our findings.

MODELING SURFACE TENSION

The continuum surface tension force (CSF) of Brackbill et al. [1] has been widely used over the past twelve years to model surface tension in multiphase flow in volume of fluid (VOF), level-set (LS) and front tracking (FT) methods. Surface tension forces acting on the interface are transformed to volume forces in regions near to the interface via delta functions, leading to ideally discontinuous interface jump conditions being modelled as smooth. Recently, ghost fluid methods (GFM) were presented in [3, 4] to impose 'sharper' boundary conditions on embedded boundaries. GFM have been employed to model surface tension in conjunction with LS techniques, since GFM require knowledge of the distance from the interface, which is the information natural for LS methods. We denote VOF-GFM as an underlying VOF method that employs GFM for surface tension, VOF-CSF as an underlying VOF method that employs the CSF method for surface tension, and LS-GFM as an underlying LS approach that employs GFM to model surface tension. VOF-GFM is an interesting alternative to LS-GFM because of the superiority of VOF methods in mass conservation over LS methods. However, since in VOF methods volume fractions are employed to track the interface, a distance function is not naturally available. We have therefore devised a novel technique [5] to reconstruct distance functions from volume fractions, allowing one to combine GFM with VOF. We now address whether or not there are advantages in using a VOF-GFM to model surface tension flows over the widely-used, classical VOF-CSF method. To aid our comparison, we consider the case of a static drop in equilibrium. The jump condition in pressure along the normal direction is $[P]=\sigma\kappa$, where σ is the surface tension coefficient and κ is the interfacial curvature. In VOF methods, a discountinous Heaviside function (volume fractions) f on a fixed grid is employed to track the interface. In cells filled with fluid 1, f is equal to 1, and in cells filled with fluid 2, f is equal to 0. For cells containing the interface, f is between 0 and 1. In VOF, a single set of governing equations are considered with an additional evolution equation to track the interface. In our case, the VOF is initialized using a recursive local mesh refinement technique for cells that contains the interface. We consider the fluid inviscid and the flow incompressible. The governing equations are:

$$\nabla.\vec{u} = 0 \quad (1), \qquad \frac{\partial(r\vec{u})}{\partial t} + \nabla.(r\vec{u}\vec{u}) = -\nabla P \quad (2), \qquad \frac{\partial f}{\partial t} + \vec{u}.\nabla f = 0 \quad (3)$$

where $\vec{u}$ is the velocity, r the fluid density assigned as $r = r_1 f + r_2(1-f)$. The above conservation equations are solved using the projection method and the VOF advection equation is solved with a PLIC (piecewise linear calculation) algorithm [6]. Next, we present how we can apply the surface tension with the CSF and GFM method. To simplify the presentation, we consider a static case with zero velocity.

Continuum Surface Tension Force (CSF) [1], [2]

In the CSF method, the surface tension force is represented as a continuum force per unit volume in regions along and near to the interface. This force $\vec{F}_S = sk\nabla f$ appears as a source term in the momentum equation. Recently a consistent formulation [2] was proposed in which the surface tension force appears in the right hand side of the pressure equation and gradients are discretized at faces:

$$\nabla.\left(\frac{1}{r_{face}}(\nabla P)_{face}\right) = \nabla.\left(\frac{s\,k(\nabla f)_{face}}{r_{face}}\right). \quad (4)$$

Ghost Fluid Methods (GFM) [3], [4]

In GFM, the pressure jump condition is applied as rigorously on the interface as the distance function f allows. The distance function is zero on the interface, negative inside the interface (fluid 1) and positive outside the interface (fluid 2). Since we are in the context of the VOF method, where we only store volume fractions, we need to reconstruct a distance function. For that we have used our reconstruction distance function (RDF) technique [5]. First, normal distances from the piecewise linear segments are computed using simple geometrical relations to all nearby cells. The piecewise linear segments are constructed based on volume fractions [6] and are part of the VOF technique. The normal distances to the nearby interfacial cells are then weighted to obtain the distance function. The weights are function of the angle made by the interfacial normal and the vector between the cell center and the linear segment centroid (the smaller the angle, the greater the contribution). In GFM, the pressure stencil is modified to apply the jump condition at the interface due to surface tension. The modifications appear in the right hand side of the pressure equation:

$$\nabla \cdot \left(\frac{1}{r_{face}} (\nabla P)_{face} \right) = F^L + F^R + F^T + F^B, \tag{5}$$

where F^L, F^R, F^T and F^B are source terms. Nonzero terms are only those at faces across which the distance function change signs. L is for left stencil (*i-1*,*i*), R for right stencil (*i*,*i+1*), T for top stencil (*j*,*j+1*) and B for bottom (*j-1*,*j*). For example, the left stencil, if $f_{i,j} \leq 0$ and $f_{i-1,j} > 0$, becomes:

$$F^L = \frac{(1/r)_{(i-1/2,j)} (s\,k)_I}{(\Delta x)^2} \tag{6}$$

Note that GFM require a series of tedious logical tests, which are not required in CSF method.

Curvature Estimation

To estimate interfacial curvature, a height function method is used, which has been found in [5] to be more accurate than techniques using a distance function or volume fraction convolution.

RESULTS

To illustrate the difference between CSF and GFM we present here the results of a static computation, with dynamic results forthcoming in the full paper. We consider a 2D drop of radius 0.25 in a unit square domain. The surface tension is 1 and the curvature is imposed to its exact value 4 for not introducing error coming from its computation. The pressure equations are solved using an iterative method Line SOR. Both methods recover the exact jump in pressure ΔP=4. However, with the CSF method, the profile is smoother than the GFM result (see Figure 1). GFM are more accurate because it takes account the exact interface position. Interestingly, both GFM and CSF results are independent of the density ratio, as can easily be shown in their formulation.

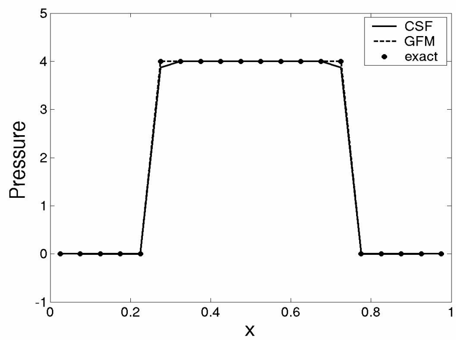

Figure 1: Pressure profile using VOF-GFM and VOF-CSF along the x direction at y=0.5. Grid is resolution 20x20.

CONCLUSIONS

We have investigated the difference between the CSF and GFM to model the jump in pressure due to surface tension at an interface. The GFM achieves a sharp jump in pressure, whereas the CSF method achieves a smooth transition. We have demonstrated that we can combine the VOF and GFM by reconstructing distance functions from volume fractions without solving an equation for the evolution of the distance function as in level-set techniques providing an alternative to the CSF method. Further scrutiny of these two methods will be made with dynamical simulations.

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