LEVEL-SET SIMULATIONS OF SHEAR FLOW WITH INERTIA PAST A DROPLET ADHERING TO A WALL WITH MOVING CONTACT LINES

Peter D. M. Spelt*

Theoretical Mechanics, University of Nottingham, Nottingham NG7 2RD, United Kingdom

Summary A level-set method for the numerical simulation of incompressible two-phase flow is developed for flows with moving contact lines. The method is used to study shear flow past a two-dimensional droplet that adheres to a solid substrate. Cases with pinned and moving contact lines can be simulated. Previous work on this problem assumed Stokes flow, whereas the present method is suitable for flow with significant Reynolds number.

INTRODUCTION

The deformation of droplets adhering to a wall in shear flow has practical applications in cleaning processes, and serves as a model of a cell adhering to a blood vessel. Of practical interest is for instance the determination of conditions beyond which part of a droplet will be sheared off. The creeping flow case with fixed contact lines has been studied thoroughly in two and three dimensions; results available for moving contact lines are restricted to creeping flow in 2D [3]. We present here results from numerical simulations for the corresponding flow with inertia, in two dimensions. For this purpose, a level-set method is adapted such that multiple contact points can be simulated.

Results are presented for cases in which the contact points are allowed to move. In order to avoid the stress singularity at moving contact lines, the usual no-slip condition is replaced by the Navier condition for the velocity component U_1 along the wall, $U_1 = \lambda \partial U_1/\partial x_2$, where the sliplength λ is small. The contact-line speed will be prescribed through $U_{cl} = \kappa(\theta - \theta_s)$, where θ and θ_s are the dynamic and static contact angle, respectively. More complicated expressions for U_{cl} (Dussan V. 1979) can be implemented in a straightforward manner.

NUMERICAL METHOD

Level set is an efficient numerical method for tracking interfaces. In [4], a level-set method is developed for the simulation of incompressible two-phase flows. In this approach, the level-set function ϕ at a point is the distance from the interface, such that an interface corresponds to a surface where $\phi = 0$. The equations of motion for both fluids involved are combined into a single continuity and momentum equation with variable density and viscosity; surface tension is represented as a sink term in the momentum equation. Required variables such as the density, viscosity and curvature are expressed explicitly in terms of ϕ. At each timestep, ϕ is advected by the fluid velocity field. To ensure that ϕ remains the signed distance function (at least close to interfaces), a redistance step is subsequently required. In order to guarentee mass conservation, the subcell fix of [2] has been implemented, and ϕ is scaled at each timestep in the way proposed in [5]. Conventional methods are used to solve the velocity and pressure fields.

Level set has been used for many flows that do not involve intersections of interfaces with solid boundaries (contact lines). In order to use level set for flows with contact lines, boundary conditions for ϕ are required, instead of using an extrapolation from the fluid interior for the value of ϕ in ghostcells. In [5], the axisymmetric spreading of oil under ice is simulated using level set (involving a single contact point). The interface is extrapolated through the wall, and the value of ϕ at ghostcells is either the signed-distance function corresponding to this extrapolated interface, or (if a normal cannot be drawn from the ghostpoint to the extrapolated interface) an extrapolation of ϕ from the fluid interior.

For more complicated problems, such as a droplet adhering to a wall, the approach in [5] must be generalized. The position of contact points is obtained by integrating the adopted expression for the contact-line speed U_{cl}. The boundary conditions for ϕ are determined as follows (Fig. 1). For points left of the left contact line shown in Fig. 1, a boundary condition (obtained from the distance to $\mathcal{L}_1$) at ghostpoints is required. For points far to the right of the right contact line, it is not possible to draw a normal to the nearest imaginary interfaces (dashed part of $\mathcal{L}_2$), the distance to be used should correspond to that from the solid part of the interface $\mathcal{L}_2$, and no boundary condition should be imposed on ϕ (the value of ϕ in ghostcells is therefore obtained from second-order extrapolation). The two points P and Q indicate possible ghost-point locations between two contact lines. At P, a boundary condition is required for ϕ, obtained from the distance to the nearest imaginary interface. At Q, the minimum distance to an interface may be that to the solid part of $\mathcal{L}_1$, which can be obtained from extrapolation of ϕ from the interior. These boundary conditions are imposed during the redistance step.

RESULTS

In Fig. 1b, the contact angles and perimeter of a droplet relaxing towards static contact angles of 120^o are shown as a function of time. The initial contact angles are 30^o, and there is no imposed shear flow. The capillary number is small: at $t = 0$, $\mu U_{cl}(t = 0)/\gamma = 3.4 \cdot 10^{-3}$ (the Reynolds number $\rho H U_{cl}/\mu = 13.3$, where $H =$ initial droplet height). Hence the drop should almost be a circular cap throughout the simulation. The perimeter is seen in Fig. 1b to compare very well

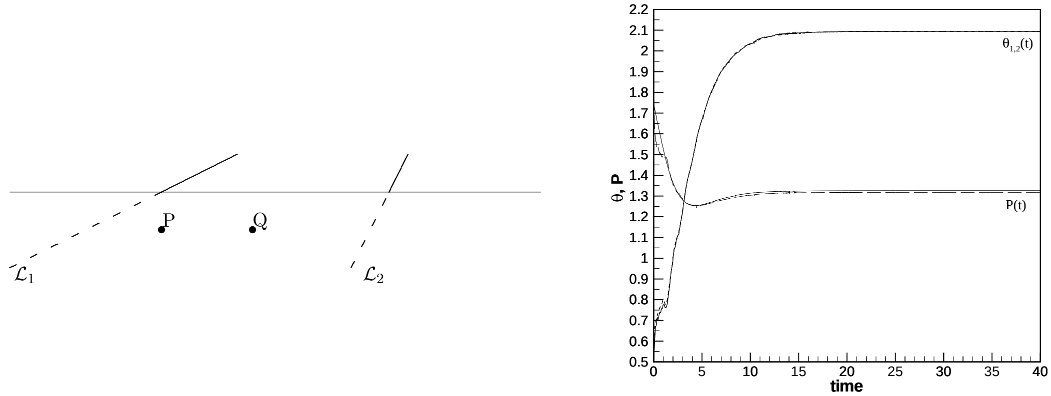

Figure 1. *(a).* Definition sketch for boundary conditions for ϕ imposed. *(b).* Droplet with moving contact lines in a fluid without external flow. Relaxation from initial contact angles of 30^o, to static contact angle of 120^o. Perimeter P and contact angles as a function of time. The long-dashed line represents the perimeter for an exact circular interface with contact angles corresponding to the values resulting from the simulations. To show the influence of slip, results for the contact angles with $\lambda = 0.01$ are indicated by short dashes.

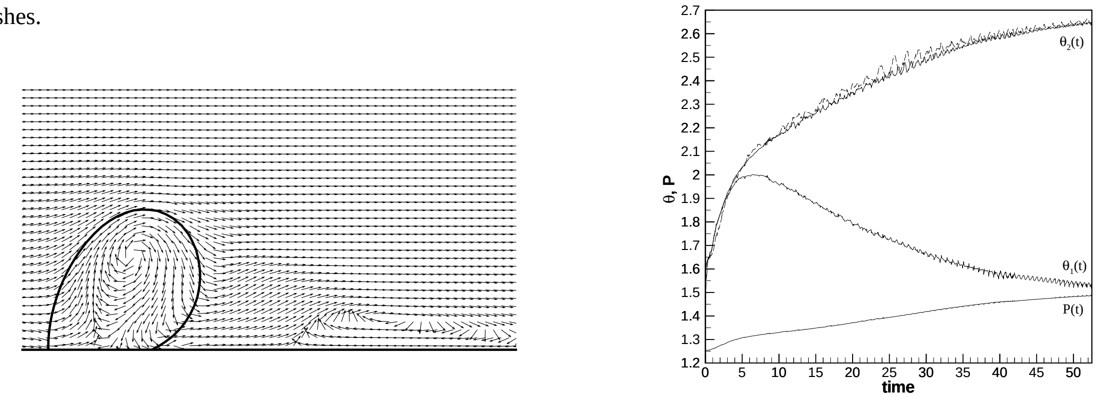

Figure 2. *(a).* Velocity flags (indicating local direction of fluid velocity) for a drop with moving contact lines in a shear flow with $\rho_1 = 1$, $\rho_2 = 20$, $\mu_1 = \mu_2 = 0.1592$, $\gamma = 3.175$. Initial contact angles are $\pi/2$; $\theta_s = 2\pi/3$. Simulations have approached a quasi-steady state, in which the drop moves at constant speed without further deformation. *(b).* Perimeter P and contact angles as a function of time, for a 128×32 mesh. The dashed line corresponds to θ_2 when using a 64×16 mesh.

with that determined from assuming a circular-cap shape and using the contact angles as a function of time. Results were also found to compare well with the boundary-element simulations of [3] for creeping shear flow past a drop on a wall.

Fig. 2 shows a droplet that is deformed by an imposed shear flow. The contact lines are allowed to move, with $\kappa = 0.1$, $\lambda = 0.01$. The droplet has approached a quasi steady state, in which it moves at almost constant speed and does not deform further. The contact angles and perimeter are shown as a function of time in Fig. 2b. A small oscillation is observed in the contact angles, that decreases with increasing mesh size. It is seen from Fig. 2a that a wake has formed downstream of the drop; this moves together with the drop. Due to the direction of rotation inside the drop and the wake, the fluid that shears just over the droplet is seen to make a U-turn between the drop and its wake; this effect was found to be especially prominent in cases of pinned contact lines, where the wake is closer to the droplet.

CONCLUSIONS AND FUTURE WORK

The level-set method proposed in [4,5] has been extended to allow for the presence of multiple contact lines that can move. The method has been used to simulate the motion and deformation of a 2D droplet adhering to a wall in a shear flow. The method is currently extended to allow for contact-line hysteresis, and is being used to determine critical values of dimensionless parameters, beyond which droplets are displaced from an adhering surface by a shear flow with inertia.

References

[1] Dussan V., E. B.: On the spreading of liquids on solid surfaces: static and dynamic contact lines. *Annu. Rev. Fluid Mech.* **11**:371-400, 1979.
[2] Russo, G. and Smereka, P.: A remark on computing distance functions. *J. Comp. Phys.* **163**:51-67, 2000.
[3] Schleizer, A. D. and Bonnecaze, R. T.: Displacement of a two-dimensional immiscible droplet adhering to a wall in shear and pressure-driven flows. *J. Fluid Mech.* **383**:29-54, 1999.
[4] Sussman, M., Almgren, A. S., Bell, J. B., Colella, P., Howell, L. H. and Welcome, M. L.: An adaptive level set approach for incompressible two-phase flows. *J. Comp. Phys.* **148**:81-124, 1999.
[5] Sussman, M. and Uto, S.: A computational study of the spreading of oil underneath a sheet of ice. *CAM Report* 98-32, 1998.