EVOLUTION OF A PAIR OF SPHERICAL BUBBLES RISING SIDE BY SIDE AT MODERATE REYNOLDS NUMBER

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Summary An extensive numerical investigation of the interaction between two spherical shear-free bubbles rising side by side in a viscous liquid has been carried out. The results show that the effect of the vorticity generated on the bubble surface changes dramatically the characteristics of the interaction process, especially by allowing the transverse force to be repulsive for low enough Reynolds number and separation distances, which results in the existence of a stable equilibrium separation distance.

Bubbly flows with moderate-to-large bubble concentration (typically from some percents to 20% in volume fraction) occur in many natural and industrial processes. However the current understanding of these flows and the models used to predict their evolution are far from satisfactory because of the difficulty in describing direct hydrodynamic interactions between bubbles, the effect of which is expected to influence greatly the spatial distribution of the dispersed phase in regions where the bubble concentration is significant. Up to now, most of the detailed theoretical investigations of these hydrodynamic interactions have been carried out either in the Stokes flow limit (following the pioneering papers by Batchelor [1],[2]) or in the potential flow approximation (eg. [3]). Nevertheless, several predictions of the latter two approaches conflict with the results displayed by finite-Reynolds-number experiments. For instance no transverse (lift) force can take place between the bubbles in the Stokes flow limit. Similarly, whereas the potential approximation predicts that when rising under the effect of buoyancy bubbly suspensions tend to organize themselves in the form of horizontal clusters ([4],[5]), experiments reveal much more homogeneous spatial distributions of bubbles.

To get some insight into the microphysics of the hydrodynamic interaction process at finite Reynolds number, we solve numerically the full Navier-Stokes equations and boundary conditions describing the three-dimensional flow past two spherical shear-free bubbles moving side by side in a viscous fluid, the distance between the two bubbles being set fixed. This is achieved by using an orthogonal boundary-fitted grid combined with a second-order accurate incompressible Navier-Stokes code based on a finite-volume discretization. Extensive tests of the influence of the grid parameters were carried out in order to make sure that the results are grid-independent and agree with potential predictions at short time, ie. before vorticity has diffused significantly from the bubble surface [6].

Using this numerical tool, we analyze the interaction between the two bubbles over a wide range of Reynolds number ($0.02 \le Re \le 500$, Re being based on the bubble diameter and rise velocity) and separation distance S ($2.25 \le S \le 20$, S being the distance between the bubbles centres normalised by the bubble radius). The computations shed light on the role of the vorticity generated at the bubble surface in the interaction process. When Re or S is large enough for the vorticity to remain confined in a boundary layer whose thickness is small compared to the distance between the two bubbles, the interaction is dominated by the irrotational mechanism that results in a maximum of the vertical velocity located in the gap between the two bubbles (Fig. 1, left). Following Bernoulli theorem, the transverse force is then attractive. In contrast, when viscous effects are large enough, the vorticity spreads out about each bubble until it is blocked in the gap by the vorticity existing about the other bubble. This interaction reduces the magnitude of the vertical velocity in the gap, resulting in a repulsive transverse force for small enough values of Re or S (Fig. 1, right). We confirm theoretically the existence of a repulsive force at low-but-finite Re by extending the asymptotic evaluation of the transverse force performed by Vasseur & Cox [7] to the case of shear-free bubbles.

When we set the separation S to a given value and vary Re, we observe that for any S the sign of the transverse force is reversed when the Reynolds number crosses a certain critical value Re_C, being attractive (resp. repulsive) for larger (resp. smaller) Re, in line with the mechanism described above. The value of Re_C depends on S and lies approximately in the range $30 \le Re_C \le 100$ for the range of separations covered by the computations. In contrast, if we set Re to a given value and vary S, different situations may happen. If Re is selected to be lower than a critical value of the order of 30, the two bubbles are found to be repelled from each other up to infinity, however large S (left part of the grey area in Fig. 2). For higher values of Re, the two bubbles are repelled only if S is either smaller than a critical value $S_1(Re)$, or larger than another critical value $S_2(Re)$, with $S_2(Re)>S_1(Re)$. To undertand this somewhat complex behavior, we consider the evolution of a pair of *freely moving* bubbles rising under the effect of buoyancy. As such bubbles have to maintain a constant vertical drag force, they cannot follow arbitrary paths in the (Re,S) plane once released with an initial separation S_i. By examining the stability of the equilibrium separation of such pairs of bubbles near the two critical curves $S_1(Re)$ and $S_2(Re)$, we show that only the first of these corresponds to a stable position. Hence, if the initial separation S_i is larger than $S_2(Re)$, the two bubbles are repelled from each other up to infinity (right part of the grey area in Fig. 2). In the opposite case $S_i<S_2(Re)$, they tend to reach the stable equilibrium configuration corresponding to $S=S_1(Re)$. As shown in Fig. 2, $S_1(Re)$ is a decreasing function of the Reynolds number which is about 3 (corresponding to a gap thickness about one bubble radius) for Re=30.

The results of this investigation demonstrate that interaction effects arising between two clean spherical bubbles rising side by side at Reynolds numbers less than about 250 (i.e. the upper limit for which deformation effects may reasonably be neglected in water) differ significantly from the picture provided by the potential flow approximation since the latter predicts that the interaction force is attractive whatever S. In particular, in contrast with the systematic formation of stable horizontal clusters observed in the potential simulations [3], [4], existence of a repulsive interaction force for small-to-moderate rise Reynolds numbers and of an equilibrium (admittedly small) separation distance for higher Reynolds numbers favours a more homogeneous spatial distribution of the bubbles. We indeed find that the interaction force is positive for *Re* larger than 50-100 (depending on S), so that horizontal clustering is likely to occur in the range $100 \leq Re \leq 250$. However the actual attractive force is significantly smaller than predicted by assuming an irrotational flow everywhere, suggesting that the clustering process requires a longer time and is less stable to external perturbations than one may infer from the potential flow theory.

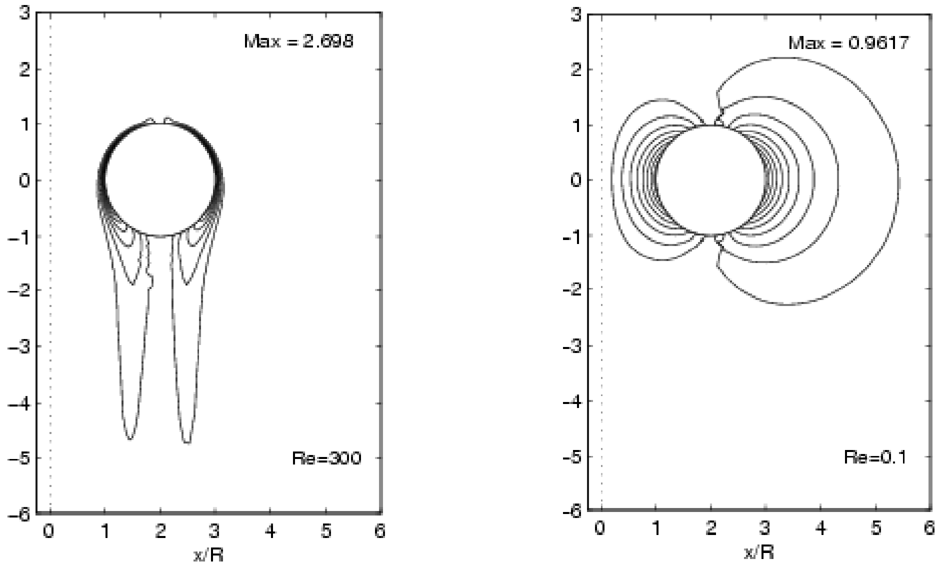

Figure 1. Iso-contours of the z-component of the vorticity in the symmetry plane z=0 for a separation S=4.
Left: Re=300; right: Re=0.1; the dotted line on the left indicates the symmetry plane between the two bubbles.

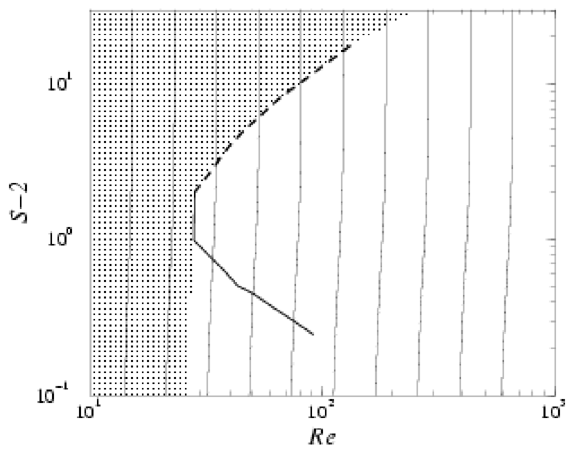

Figure 2. Evolution of the equilibrium separation of a pair of freely rising bubbles in the (Re, S) plane.
——$S_1(Re)$; ------- $S_2(Re)$; ——iso-drag curves; the grey area indicates the sub-region of initial separations S_i for which the bubbles subsequently separate up to infinity, while the white area is the basin of attraction of the stable equilibrium position $S_1(Re)$.

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