VISCOUS EXTENSIONAL FLOW AND DROP BREAK-OFF UNDER GRAVITY

Ernest O. Tuck*, Yvonne M. Stokes*
Applied Mathematics, The University of Adelaide, Australia 5005

Summary
This paper concerns finite drops of very viscous Newtonian fluids falling under gravity in an extensional flow, starting from rest in contact with a solid boundary. Emphasis is on the role of initial conditions, together with balances between forces such as inertia, gravity, viscosity and surface tension. Under gravity and viscosity alone, break-up occurs in finite time, but inertia makes that time formally infinite, and surface tension will further modify this conclusion. A slender-filament theory is used to illustrate these effects.

INTRODUCTION

We seek further understanding of the mechanisms governing gravity-driven extensional flows of viscous liquids which, like honey dripping from an up-turned spoon, exhibit elongation, necking to form a drop suspended by a thin filament and, finally, pinch-off of the drop and further breakup of the filament.

Extensional flow is seen in the dripping of a tap and web spinning by spiders and insects, as well as in important technologies such as ink-jet printing, polymer and glass fibre spinning, blow moulding, soldering and rheological measurement.

Ink-jet printing in particular has driven much recent research in this area, and continues to do so [1,2]. This technology is not only used in the well-known ink-jet printer, but also in biochip arraying and fabrication of transistors [1]. The size and shape of drops impacts on printing quality, as does the formation of satellite drops.

An application area of particular interest is industrial glass moulding, where the first stage is the gravity-driven formation of a drop of molten glass. The size and shape of this drop can be important, as in the production of large CRT screens where it affects the quality of the screen pressed from it [5]. In blow moulding of containers, elongational stretching due to gravity, and the resulting necking of the glass, may lead to unacceptable non-uniformity of container wall thickness. Similarly, in fibre spinning, pulling of the fibre may lead to a non-uniform cross-section.

Extensional flows and the mechanisms which cause breakup into drops have long been of scientific interest [2]. Development of theoretical ideas to explain the observations and computer codes to simulate the free-surface flows has been relatively slow, much of the literature appearing in the last two decades.

By far most attention in the literature has been given to surface-tension driven pinch-off of a Newtonian liquid drop and the subsequent formation of satellite drops. One-dimensional approximations are common [3] and, since the work of Keller and Miksis [4] and Peregrine et al. [6], much focus has been on finding similarity solutions in the neighbourhood in space and time of the pinch-off, motivated by the experimental observation that breakup of a fluid thread appears in some cases to be largely independent of initial conditions, but strongly dependent on fluid properties such as viscosity [7]. The extent of dependence of break-up behaviour on initial conditions is however still an open problem that has been little probed.

This study concerns high-viscosity gravity-driven drops and filaments before pinch-off, when initial conditions and forces other than surface tension do influence the flow. For highly viscous Newtonian fluids subject to an external pull (e.g. smooth honey dripping under gravity), very long filaments can develop and persist well beyond breakup times that would be predicted by current surface-tension dominated theories. Hence “this remains an extremely interesting problem to be studied in more detail” [2]. Previous research by the present authors ([8, 9] shows pinch-off features even with surface tension neglected, and has led us to make the suggestion that, at least for very viscous Newtonian fluids, inertial and viscous forces rather than surface tension forces are dominant in the early stages of pinch-off, with surface tension playing the dominant role only later when the thread diameter becomes very small. The present paper will test this suggestion by explicit inclusion of surface tension in the models.

The present research is innovative in taking a less surface-tension dominated approach than that of most other researchers in this area. Studies neglecting both surface tension and inertia are reported in [8] and [10]. In a recent [9] extension of the work of [8], we have included inertia, while still for the time being neglecting surface tension. Our new results then show flow features not previously recognised that we believe provide a pre-cursor to drop pinch-off, with actual pinch-off occurring later through the action of surface tension, once the local length scales become sufficiently small. Most interestingly, one of these features is some initial pinching, which is usually but perhaps erroneously attributed to the action of surface tension. This behaviour is not properly understood, and calls for a more detailed investigation.

SUMMARY OF SLENDER-DROP THEORY

Although we have also used a finite-element direct solution of the axisymmetric Navier-Stokes equations for drops of arbitrary initial profile, in the present paper we discuss only a one-dimensional approximate theory for slender drops. That is, we assume that initially the fluid occupies a region which is of small width relative to its length, and is at rest in contact with and beneath a given boundary, here taken as a plane horizontal wall. The subsequent motion will then be dominated by the downward velocity component, and will be of an extensional nature.

In the absence of surface tension, the one-dimensional Lagrangian equations of motion can be written [9]

$$X(\xi,t) = \int_0^{\xi} \frac{A_0(\xi_1)}{A(\xi_1,t)}\, d\xi_1$$

and

$$A(\xi,t) = A_0(\xi) - \frac{\rho}{\mu^*} \int_{\xi}^{L_0} A_0(\xi_1)\,[gt - X_t(\xi_1,t)]\, d\xi_1$$

where $x = X(\xi,t)$ is the position at time t of the fluid particle that was at $x = \xi$ at time $t = 0$, and $A(\xi,t)$ is the section area of the drop, starting at $A_0(\xi) = A(\xi,0)$ with initial length L_0. The term "X_t" in the second equation represents inertia, neglected in [8], and $\mu^* = 3\mu$ is the extensional or Trouton viscosity, g is the acceleration of gravity in the positive x-direction, and ρ is (constant) density.

If the inertia term is neglected, these equations provide an explicit solution by quadratures for any initial shape $A_0(\xi)$. This solution in general exhibits finite-time blow-up, with length $L(t) = X(L_0,t) \to \infty$ at a finite time $t = t^* \propto \mu^*/(\rho g L_0)$, the constant of proportionality depending on the initial shape $A_0(\xi)$. At the same time, the section area $A(\xi,t)$ goes to zero at some station $\xi = \xi^*$ (which is often at the wall $\xi^* = 0$), a signal that break-off will occur at that time and place. This conclusion is similar to that of Wilson [10] for a viscous fluid dripping slowly out of a capillary tube.

If inertia is retained, the problem can be recast as a nonlinear diffusion equation for $X(\xi,t)$, and numerical solutions were given in [9]. These solutions have the property that it takes an infinite time for the length $L(t)$ to approach infinity, but for large effective Reynolds number $\rho^2 g L_0^3/{\mu^*}^2$, there is a very rapid increase in extension close to time $t = t^*$. There is no actual break-off of the drop, the area $A(\xi,t)$ remaining positive for all ξ and t. However, $A(\xi,t) \to 0$ as $t \to \infty$, at the particular station $\xi = \xi^*$ where the inertia-less drop would have broken at $t = t^*$. Meanwhile the remainder of the drop approaches free fall with acceleration g. Thus as $t \to \infty$, an almost rigid drop forms, together with an ever-thinning filament connecting it to the original boundary. Because of neglect of surface tension, the shape of the final freely falling drop depends critically (and only) on the initial shape $A_0(\xi)$.

We now propose to include surface tension in this model and results will be reported at ICTAM04. Such an inclusion is necessary when the filament diameter becomes comparable to the meniscus scale, which will certainly happen very near to the time of drop pinch-off, if not before. We expect that these results will show a continuous transition from the drop-formation history described above, toward the usual one dominated by surface tension for large time, where the connecting filament has actually broken, and the free drop is asymptotically spherical.

CONCLUDING REMARKS

Our studies [9] including inertia and neglecting surface tension also revealed some interesting and unexpected features of the flow that require further investigation, especially with surface tension included in the model. First, both numerical (finite-element) direct solutions of the Navier-Stokes equations for drops of arbitrary initial aspect ratio, and semi-analytic solutions of the above one-dimensional equations for initially-slender drops show that the acceleration exceeds g for a period of time at a point in the interior of the drop. Second, finite-width numerical simulations show the beginnings of drop pinch-off at this point, which is in the transition region between the main drop and the filament. In the absence of surface tension, this incipient pinch-off is surprising and leads us to suggest that initial conditions along with viscous and inertial forces initiate pinch-off and dictate its position, with surface tension taking over as the dominant force only later, when the local length scales become very small, comparable to the meniscus scale, due to this pinching.

ACKNOWLEDGEMENT

This research is supported by the Australian Research Council.

References

[1] A.U. Chen and O.A. Basaran, A new method for significantly reducing drop radius without reducing nozzle radius in drop-on-demand drop production, *Phys. Fluids* (2002) 14(1), L1–L4.
[2] J. Eggers, Nonlinear dynamics and breakup of free-surface flows, *Rev. Mod. Phys.* (1997) 69(3), 865–929.
[3] F.J. García and A. Castellanos, One-dimensional models for slender axisymmetric viscous liquid jets, *Phys. Fluids* (1994) 6(8), 2676–2689.
[4] J.B. Keller and M.J. Miksis, Surface tension driven flows, *SIAM J. Appl. Math.* (1983) 43(2), 268–277.
[5] R.M.M. Mattheij, Personal communication (2001).
[6] D.H. Peregrine, G. Shoker and A. Symon, The bifurcation of liquid bridges, *J. Fluid Mech.* (1990) 212, 25–39.
[7] X.D. Shi, M.P. Brenner and S.R. Nagel, A cascade of structure in a drop falling from a faucet, *Science* (1994) 265, 157 & 219–222.
[8] Y.M. Stokes, E.O. Tuck and L.W. Schwartz, Extensional fall of a very viscous fluid drop, *Q. Jl. Mech. appl. Math.* (2000) 53(4), 565–582.
[9] Y.M. Stokes and E.O. Tuck, The role of inertia in extensional fall of a viscous drop, *J. Fluid Mech.*, (2004) 498, 205–225.
[10] S.D.R. Wilson, The slow dripping of a viscous fluid, *J. Fluid Mech.* (1988) 190, 561–570.