CONVECTIVE PHENOMENA IN ROTATING ANNULI HEATED ON PERIPHERY

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Summary Results of numerical simulation of 2D and 3D time-dependent free convection in rotating annular air-filled cavities heated on the periphery and cooled at the inner radius are presented. The cavity radial aspect ratio is equal to 0.35, the centrifugal Rayleigh number is ranged from $2 \cdot 10^4$ to $5 \cdot 10^7$. In the middle of this *Ra*-range, stable flow regimes with two, four or six vortices can be obtained with the 2D formulation for an unlimited annulus. In a 3D configuration, with two bounding adiabatic discs, the two-vortex regime is inhibited.

INTRODUCTION

Rapidly rotating cavity heated on the periphery are typical elements of gas turbine rotors. A sealed annular cavity formed by two discs and two cylindrical walls is considered as a generic configuration for studies of convective phenomena induced by centrifugal buoyancy [1]. An experimental study of free convection with a radial heat flow in rotating air-filled cavities was performed by Bohn et al. [2] at the Rayleigh numbers exceeding 10^7. Theoretical analysis of buoyancy-induced convection in rotating cavities faces the difficulties associated with three-dimensional and unstable behaviour of the flow, similar to that one has for the Rayleigh-Bénard convection in a fluid layer confined between two horizontal plates, the lower of which is hotter than the upper one. Numerical simulation of 3D convective phenomena in rotating cavities heated on the periphery is very time-consuming. Known attempts were limited by cases of segmented cavities with sectors of 45 degrees [2] or 60 degrees [3]. Note that in the case of segmented cavities the radial partitions stabilise the convection and result in a considerable reduction of the computational grid size. A simplified 2D formulation of the buoyancy-induced convection, assuming rotating annuli are unlimited in the axial direction, were used in [4] for a numerical analysis based on the Boussinesq's approximation. The present contribution covers a numerical analysis of time-dependent air convection flows induced by the centrifugal buoyancy both in an annulus unlimited in the axial direction and in an annular cavity bounded by two adiabatic discs.

STATEMENT OF THE PROBLEM AND COMPUTATIONAL ASPECTS

For the present 3D numerical simulation, following the experimental study [2], we have chosen a rotating annular cavity with the radial ratio, r_i / r_o, equal to 0.352, and the axial ratio, s / r_o, equal to 0.338. Here r_o is the outer radius of the cavity, and s is the cavity axial size. The Rayleigh number is defined as $Ra = Pr\, r_m \Omega^2 (\Delta r)^3 T_m^{-1} (T_o - T_i) / \nu^2$, where Ω is the angular velocity, $T_m = (T_o + T_i)/2$, $r_m = (r_o + r_i)/2$, $\Delta r = r_o - r_i$. According to [2], we set the temperature of the inner cylinder, T_i, equal to 283K, and the outer cylinder temperature, T_o, equal to 373K. The air pressure level is taken as 1 bar. The Prandtl number, *Pr*, is set to 0.71.

We consider also the 2D formulation of the problem, assuming the annulus of $r_i / r_o = 0.352$ is unlimited in the axial direction ($s \to \infty$). The thermal conditions on the cylindrical walls are kept the same as in the 3D case.

The computations on the base of the full Navier-Stokes equations and the energy equation written with the low-speed flow approximation were performed using the well-validated in-house code SINF, which is under long-time development at the Department of Aerodynamics of the St.-Petersburg State Polytechnic University. This advanced 3D Navier-Stokes solver is based on the second-order finite-volume spatial discretization using the cell-centred variable arrangement and body-fitted block-structured grids. A low-dissipative scheme blending the fourth-order central and the second-order upwind techniques is used for convective flux evaluation. The discretization of time derivatives is done with a three-time-level, second-order implicit scheme. In the case of low-speed flows, the artificial-compressibility method is applied at each time step. Additional details of the solver can be found elsewhere [5].

Results presented were obtained using a non-uniform grid consisting of 56×144 cells in the rotation plane and 56 cells in the axial direction (for the 3D analysis). The grid was clustered both to the cylindrical walls and to the bounding discs.

RESULTS AND DISCUSSION

Multiple regimes of 2D convection

Parametric computations on the base of the 2D formulation were performed for the Rayleigh number ranging from $2 \cdot 10^4$ to 10^8. It has been established that at the Rayleigh numbers ranged from (approximately) $2 \cdot 10^4$ to 10^5 one is

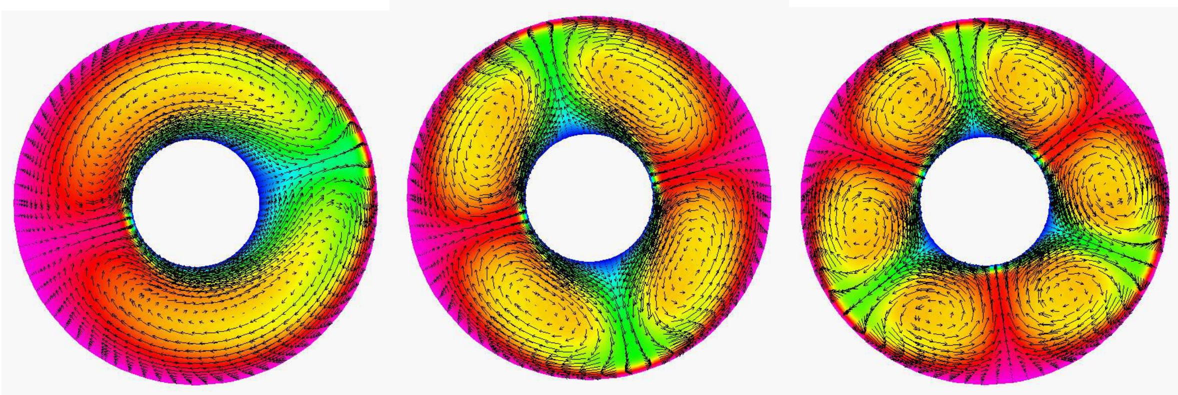

Fig. 1. Three possible regimes of buoyancy-induced 2D convection in an air-filled rotating annulus heated on periphery. Results of unsteady computations with a grid of 56×144 cells at $Ra = 10^5$: rarefied patterns of velocity vectors are imposed on color maps of temperature distributions (scales are common for all the regimes).

able to obtain three stable convection regimes that differ in the number of large-scale vortices. The first regime is characterized by the two-vortex structure of the convection, one of the vortices is cyclonic, and the other is anticyclonic. Respectively, for the second and the third regime, four-vortex and six-vortex convection patterns are observed, as illustrated in Figure 1 for $Ra = 10^5$. The vortices can drift in the circumferential direction. One or another regime develops depending on initial velocity and temperature fields used for a computational run. The two-vortex regime has the narrowest attraction area. In fact we were able to obtain this regime by an artificial way only, generating an initial two-vortex structure via solving the problem of gravity-induced convection in a horizontal annulus. The four-vortex regime exists at the Rayleigh numbers ranged from (approximately) $2{\cdot}10^4$ to 10^7, and solutions for the six-vortex regime can be obtained for the whole *Ra*-range considered.

Specifics of 3D convection

Results of the 3D numerical analysis performed have led to the conclusion that flow phenomena near the bounding discs suppress the two-vortex regime. This regime degenerates even if initial fields are obtained by translation of a 2D two-vortex solution in the axial direction. Possible regimes with four-vortex or six-vortex large-scale structures manifest a pronounced chaotic behavior at $Ra>10^6$, so that the unsteady convection becomes a “conglomeration” of ordered structures and chaotic components. In the middle plane, between the discs, the flow patterns are similar to those obtained with the 2D approach. Near the discs one can observe formation of the Ekman-type layers [1] which are permanently disturbed by perturbations coming from the corner regions. The vortices are in a relative circumferential motion (drift) and the direction of this drift is opposite to the direction of rotation. A whole turn of the vortex structure takes approximately 230 periods of rotation.

Heat transfer data

Computational data obtained for the heat rate transported from the outer to the inner cylinder show that the Nusselt numbers, *Nu*, for 2D four-vortex convection regimes are by about 25% higher than those for the two-vortex convection, if the latter exists, and by 5 to 15% lower than *Nu*-values in the six-vortex case. For the 3D configuration, the Nusselt numbers are by about 15% lower than those for correspondent 2D solutions. So, one may conclude that the 2D formulation is a fruitful approach for express evaluations of heat transfer in rotating cavities bounded by adiabatic discs, at least for the *Ra*-range examined. Note finally that at $Ra > 10^5$ the Nusselt numbers computed for the six-vortex regimes are in a good agreement with values evaluated with the correlation, $Nu \sim Ra^{0.228}$, given in [2].

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