MULTIPLICITY OF PATTERNS IN CYLINDRICAL CONVECTION

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Summary The experiments of Hof, Mullin and Lucas on Rayleigh-Bénard convection in a cylindrical system are simulated numerically using a pseudospectral three-dimensional code. We confirm that for their parameter values, there exist multiple stable solutions. Starting from a perturbed conductive state, we obtain different final patterns, depending on the Rayleigh number. We then use these flows to initialize the simulations for other Rayleigh numbers. In this way we obtain many different stable solutions for the same Rayleigh number – two, three or four parallel rolls, a three-spoke pattern and even an axisymmetric state.

CYLINDRICAL RAYLEIGH-BÉNARD CONVECTION

The Rayleigh-Bénard instability in a fluid layer heated from below and cooled from above is one of the most ancient and well-known prototypes of pattern formation. When the horizontal dimensions are comparable to the height of the fluid, the shape of the container plays a crucial role in determining the patterns formed. In a vertical cylinder, the Rayleigh number thresholds for onset of convection from the motionless conductive state have been already well described [1]. Secondary bifurcations leading to other patterns have been studied numerically for a few cases [2, 3, 4, 5]. Experiments by Hof *et al.*[6] produced a large number of convective patterns by increasing and decreasing the Rayleigh number in a variety of ways. For example, five different patterns could be obtained at $Ra = 14200$. Our aim is to reproduce, understand, and extend these results.

NUMERICAL SIMULATION

In our numerical code [7], the Navier-Stokes equations are integrated by a classical pseudospectral method, where the velocity and temperature fields are represented using Chebyshev polynomials in the radial and vertical directions and Fourier series in the azimuthal direction. The number of gridpoints or modes is 36 in the radial, 80 in the azimuthal and 18 in the vertical direction. The time-stepping scheme is the Adams-Bashforth formula for the nonlinear term and Crank-Nicolson formula for the linear term. An influence matrix method was used to impose incompressibility.
Matching the parameters in [6], we set the Prandtl number $Pr \equiv viscosity/thermal\ diffusion$ to 6.7 (that of water) and the aspect ratio $\Gamma \equiv radius/height$ to 2.0. We have performed a sequence of simulations, varying the initial state and the Rayleigh number, in order to find the asymptotic state for each configuration.

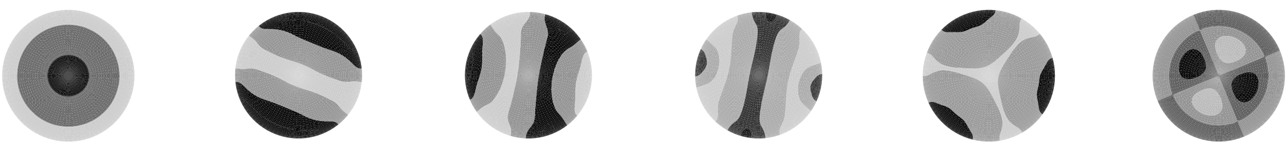
RESULTS

Sudden start from a quasi-conductive state

We initialized the simulations with a perturbed conductive solution satisfying the boundary conditions. Depending on the Rayleigh number, this state evolves towards different flows – see the left part of the diagram on fig. 2.
For $Ra \lesssim 1900$, the initial perturbation decays to zero, resulting in the conductive state. For Ra near 2000, the final state is a convective state with symmetry D_2, which we will call the 'pizza state' (see fig. 1f). For Ra between 3000 and 9000, the system evolves towards a four-roll state. For Rayleigh numbers between 10000 and 20000, the final solution is always a three-roll state, whose roll boundaries become thinner and more curved with increasing Ra. Finally, for $Ra \approx 23000$, the final pattern consists of three radial spokes that we will refer to as the 'mercedes pattern'.

Evolution from convective states

We now use the convective states described above as initial states and change the Rayleigh number. The results of the simulations are represented on fig. 2. For example, we used the mercedes pattern obtained for $Ra = 23000$ as an initial field at $Ra = 15000$. The resulting final solution was still of mercedes form. In other words, the Rayleigh number



(a) torus (b) 2 rolls (c) 3 rolls (d) 4 rolls (e) mercedes (f) pizza

Figure 1. Stable steady flows obtained by the nonlinear simulation. Temperature variation on a horizontal slice at mid-height of the cylinder. Dark areas represent hotter ascending fluid and bright areas colder descending fluid. For each flow, there also exists an equivalent form, in which the signs of the temperature variation and vertical velocity are reversed.

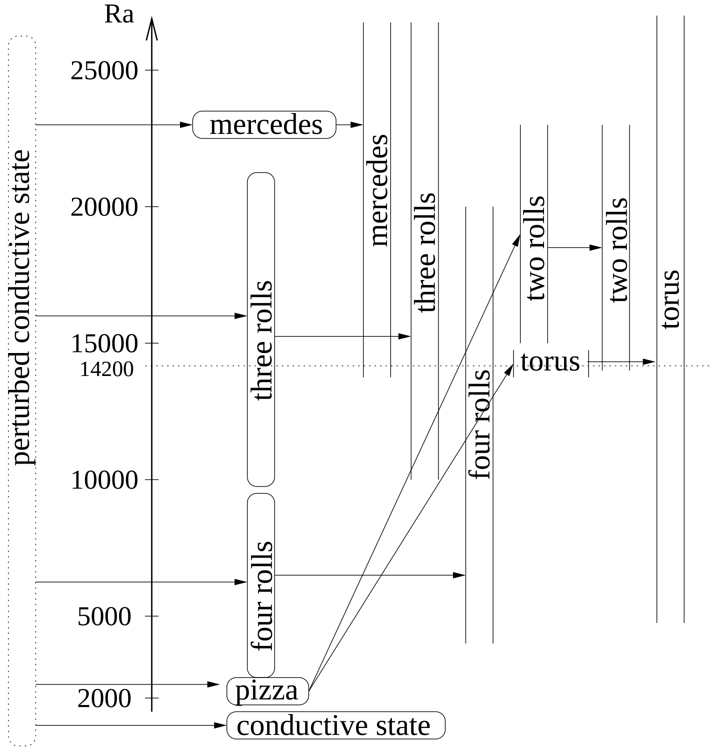


Figure 2. Schematic diagram of the dependence of the final flow pattern on Rayleigh number and on initial state. A first series of simulations used a perturbed conductive state as initial condition over a range of Ra to obtain the patterns listed in the first column. A second series used these patterns as initial conditions, altering the value of Ra. For $Ra = 14200$ we obtain five different final states, including an axisymmetric flow.

range over which the mercedes pattern remains stable is much larger than the range over which this pattern is obtained via evolution from the perturbed conductive state. The same is true for the three-roll and four-roll states. We have not ascertained the stability range for the pizza pattern. However, a large increase of Ra above 15000 leads to a transition to a two-roll state and, interestingly, for $Ra \approx 14000$, to an axisymmetric state consisting of one toroidal roll. This toroidal state remains stable over a very large Rayleigh-number range.

CONCLUSION

We have successfully simulated numerically all of the steady patterns obtained experimentally by Hof. For the same Rayleigh number ($Ra = 14200$) we observed five stable steady solutions: a toroidal roll; two, three and four parallel rolls; and a three-spoke ('mercedes') pattern. An additional 'pizza' pattern is observed at lower Ra. Despite the great variety of flows we have obtained, we are far from an exhaustive study even for this specific configuration of control parameters. It is very likely that other stable solutions exist which we did not observe because they were topologically too far from any of our initial conditions. We also found many transitional patterns that should be described in the future.
There is still much work to be done in classifying the patterns and determining their exact stability limits. Our nonlinear simulations should be refined by a linear analysis, in order to insure that a long-lived pattern is really asymptotically stable. Finally, we hope to construct a bifurcation diagram detailing and organizing the various patterns.

Computations were performed on the NEC-SX5 at the IDRIS-CNRS supercomputer center, project 1119.

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