NONCLASSICAL DYNAMICS OF LAMINAR DENSE GAS BOUNDARY LAYERS

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Summary While inviscid flows of dense gases have been studied intensively in the past, viscous effects have received much less attention so far. It is the aim of the present study to show that the observed unconventional gasdynamic behaviour of such gases may strongly influence the behaviour of laminar boundary layers. Specifically it is found that boundary layer separation may be delayed in decelerating flows while acceleration of the external flow may cause a substantial reduction of the wall shear.

GASDYNAMIC PROPERTIES OF DENSE GASES

In the last 15 years the flow properties of dense gases, i.e. gases in the general neighbourhood of the thermodynamic critical point, have received increasing interest. Studies in which thermo-viscous effects where neglected have shown that such flows may lead to a number of new and unexpected phenomena provided that the molecular complexity of the medium under consideration is sufficiently high, see for example [1]. These effects can be inferred from the behaviour of a single thermodynamic quantity, the so-called fundamental derivative

$$\Gamma = \frac{\tilde{v}^3}{2\tilde{c}^2}\left(\frac{\partial^2 \tilde{p}}{\partial \tilde{v}^2}\right)_{\tilde{s}} . \qquad (1)$$

Here $\tilde{c}, \tilde{p}, \tilde{v} = 1/\tilde{\rho}$ and $\tilde{s}$ denote, respectively, the speed of sound, the pressure, the specific volume and the entropy. According to equation (1) Γ characterises the curvature of isentropes in the $\tilde{p}, \tilde{v}-$diagram. In the case of perfect gases $\Gamma \geq 1$. However, Γ may assume values less than one or even negative values in fluids having large isochoric heat capacity $\tilde{c}_v$ relative to the gasconstant $\tilde{R}$. Fluids with this property are now commonly termed BZT-fluids in recognition of the pioneering work of Bethe, Zel'dovich and Thompson and exhibit a number of unconventional gasdynamic properties. For example, differentiation of the local Mach number M with respect to $\tilde{v}$ at constant $\tilde{s}$ yields the result

$$\frac{1}{M}\frac{dM}{d\tilde{v}}\bigg|_{\tilde{s}} = \frac{1}{\tilde{v}}\left[\frac{1}{M^2} + \Gamma - 1\right] . \qquad (2)$$

If $\Gamma \geq 1$ the expression on the right hand side is a strictly positive quantity and, consequently, M increases monotonically with increasing values of $\tilde{v}$. In contrast, regions of $\Gamma < 1$ in the $\tilde{p}, \tilde{v}-$diagram may lead to a non monotonous Mach number dependency on $\tilde{v}$. Here we are interested if and how this nonclassical gasdynamic effect influences the behaviour of laminar boundary layers.

LAMINAR BOUNDARY LAYER FLOWS

Specifically we consider flows past an adiabatic flat plate and assume that the streamwise velocity component $\tilde{u}_e$ varies linearly with increasing distance $\tilde{x}$ from the leading edge. Using suitable defined non-dimensional quantities this can be expressed in the form

$$u_e = 1 + \alpha x, \quad \alpha \pm 1. \qquad (3)$$

Linearly retarded flows of dense gases, $\alpha = -1$, have been considered first in [2]. Similar to incompressible flows and

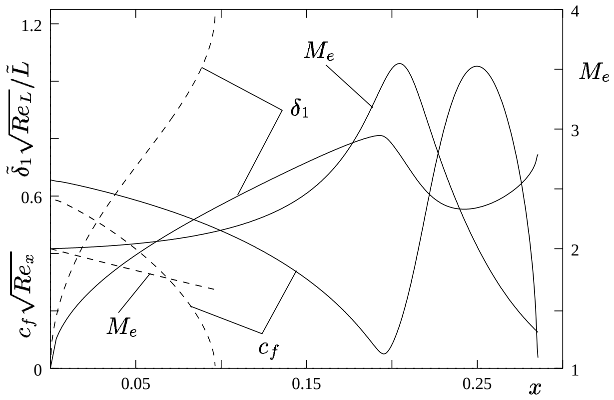

Fig. 1: Distributions of c_f, M_e and δ_1 for $\tilde{\rho}_\infty = 0.2\tilde{\rho}_c$, $\tilde{T}_\infty = 1.001\tilde{T}_c$, $M_\infty = 2$ and $\alpha = -1$.
- - - N_2 —— FC-71

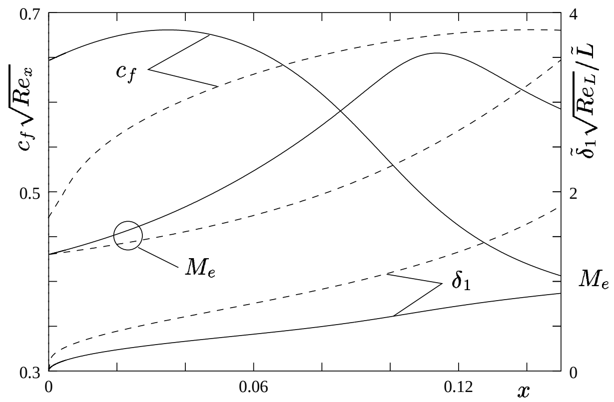

Fig. 2: Distributions of c_f, M_e and δ_1 for $\tilde{\rho}_\infty = 1.4\tilde{\rho}_c$, $\tilde{T}_\infty = 1.03\tilde{T}_c$ and $M_\infty = 1.3$ and $\alpha = 1$.
- - - N_2 —— FC-71

compressible flows of ideal gases, e.g. [4], the boundary layer development in dense gases with small or moderately large heat capacities is found to terminate in a separation singularity. An example is provided by Nitrogen N_2 with freestream conditions $\tilde{\rho}_\infty = 0.2\tilde{\rho}_c, \tilde{T}_\infty = 1.001\tilde{T}_c, M_\infty = 2$ at $x = 0$. Here $\tilde{\rho}_c$ and $\tilde{T}_c$ denote the values of the density and the temperature at the thermodynamical critical point. Calculations based on the Martin-Hou equation of state which is able to capture dense gas effects, e.g. [1], then yield $\Gamma_\infty = 2.2$ and $\Gamma > 1$ in the whole computational domain. As a consequence, decreasing values of u_e are associated with decreasing values of the Mach number M_e in the external inviscid flow region. The resulting pressure rise causes the local friction coefficient c_f to decrease also and the wall shear stress distribution eventually tends to zero in an irregular manner while the displacement thickness $\delta_1 = \tilde{\delta}_1/\tilde{L}$ grows without bound, Fig. 1. Here Re_x and Re_L denote Reynolds numbers formed with the distance $\tilde{x}$ from the leading edge and the reference length $\tilde{L}$, respectively.

Also included in Fig. 1 are results for the BZT-fluid FC-71 having the same freestream conditions as N_2. In contrast to Nitrogen, however, $\Gamma_\infty = 0.71 < 1$ and M_e, therefore, increases rather than decreases with growing distance x initially. How this affects the boundary layer development can be inferred most easily from the gasdynamic equation holding in the outer inviscid flow region. It predicts that decelerating supersonic flows of the form (3) generate negative normal velocity components which increase in magnitude as the distance from the plate becomes larger. This counteracts the effect resulting from the boundary layer displacement and may cause a reduction of the associated momentum outflux leading in turn to a delay of separation as observed in Fig. 1. Moreover, since the phenomenon of "negative displacement"intensifies with increasing values of M_e one expects that the displacement body felt by the external inviscid flow may shrink rather than expand in the flow direction if M_e is sufficiently large. This is confirmed by the numerical solution for FC-71. The associated momentum flux towards the wall then is able to overcome the onset of separation and c_f rises sharply. Eventually, however, Γ leaves the range of values necessary for a Mach number increase. M_e then starts to drop which in turn quenches this effect. As a consequence, c_f drops also and the formation of a separation singularity is inevitable. Nevertheless, the results plotted in Fig. 1 clearly show that dense gas effects may be utilised to delay boundary layer separation in BZT-fluids.

A NEW FORM OF MARGINAL SEPARATION

If the freestream density is slightly reduced while the temperature and the Mach number are kept fixed the minimum in the wall shear stress distribution is found to decrease and one finally obtains the limiting case $\tilde{\rho}_\infty = \tilde{\rho}^*$ where the wall shear vanishes in a single point but immediately recovers. At the point of vanishing wall shear the displacement body is found to exhibits a sharp corner. As a consequence, classical boundary layer theory fails locally but this failure can be overcome by applying an interaction strategy which then also allows to account for the occurrence of short separated flow regions. To this end the full Navier-Stokes equations are analysed in the double limit $Re_L \to \infty$, $\tilde{c}_v/\tilde{R} \to \infty$ using asymptotic methods. This results in a three layer structure of the local interaction region and an integro-differential equation for the scaled wall shear known from the theory of marginally separating incompressible flows, e.g. [3]. However, the controlling parameter a entering the equation now has a completely different meaning

$$a \propto Re_L^{2/5} \frac{\tilde{\rho}_c\,(\tilde{\rho}^* - \tilde{\rho}_\infty)}{\tilde{\rho}^* \tilde{\rho}_\infty}. \tag{4}$$

Also it should be noted that the physical mechanism causing the boundary layer to overcome the onset of separation is new. Marginal separation here occurs in a strictly decelerated flow which is possible only due to the non-monotonic Mach number variation.

CONCLUSIONS

Applications of the considerations concerning the external flow field associated with (3) to cases of favourable pressure gradient, $\alpha = 1$, suggest that the phenomenon of non monotonous Mach number variation may have a negative effect on the boundary layer evolution by reducing the wall shear stress. Again this is confirmed by numerical calculations as shown in Fig. 2 where, similarly to the case of linearly retarded flows depicted in Fig. 1, results for the regular fluid N_2 and the BZT fluid FC-71 are compared. In summary one therefore concludes that dense gas effects can effectively be exploited to delay or even avoid separation under the action of adverse pressure gradients but may drive the flow towards separation if a favourable pressure gradient is imposed. This is certainly of basic scientific interest but of importance also in engineering applications.

References

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