WAVE BREAKING AND EQUILIBRIUM SURFACE WAVE SPECTRA

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Summary Breaking waves enhance energy and momentum fluxes from wind to surface waves. The breaking wave effect on the form of the equilibrium range of surface wave spectra is investigated. The theory is based on the conservation of momentum and energy in the wave boundary layer together with the conservation of the wave action spectrum. The results show that the breaking wave effect is not very strong for fully-developed wave spectra but may be significant for the spectra of growing seas.

INTRODUCTION

Under given wind forcing the spectra of ocean surface gravity waves attain an equilibrium state at frequencies much higher than the peak frequency. Knowledge of the equilibrium range of the wave spectrum is of practical importance since existing numerical wave prediction models cannot solve right down to the smallest gravity waves. Instead they resolve explicitly only to a fixed maximum wavenumber and then patch on to the spectrum a 'tail' to represent the equilibrium range. Therefore these models need to know the solution for the equilibrium range before being integrated. Although many observational and theoretical studies were performed to investigate the equilibrium spectra, there remain discrepancies in the form of the equilibrium spectra among different studies.
The evolution of surface waves are described in terms of the spectral density of wave action $N(\vec{k})$,

$$\frac{dN}{dt} = -\nabla_k \cdot T(\vec{k}) + S_w - D \qquad (1)$$

where $\vec{k}$ is the wavenumber vector, S_w is the wind input, $T(\vec{k})$ is the flux of the wave action by nonlinear wave interactions (∇_k is the gradient operator in $\vec{k}$) and D is the dissipation due to wave breaking. Within the equilibrium range the sum of the three forcing terms must vanish. Phillips [1] assumed that in the equilibrium range the three input terms to the wave action conservation equation are all of the same order of magnitude and balance one another, and that the divergence of the wave action flux is proportional to the cube of the local wave spectrum. Phillips [1] further assumed that the wave growth rate is determined by the total wind stress τ_{tot} for the entire equilibrium range, and predicted that the equilibrium wavenumber spectrum is proportional to $k^{-7/2}$ ($k = |\vec{k}|$) and the friction velocity u_*.
Hara and Belcher [2] extended Phillips' [1] theory by accounting for the overall conservation of momentum. Waves exert a drag on the airflow so that they support a fraction of the applied wind stress, which thus leaves a smaller turbulent stress near the surface to force growth of shorter wavelength waves. Formulation of the momentum budget accounting for this *sheltering* constrains the overall conservation of momentum and leads to a local turbulent stress that reduces as the wavenumber increases. This local turbulent stress then forces wind-induced wave growth. Hara and Belcher [2] predicted that the equilibrium degree of saturation is expressed in a simple analytical form:

$$B(k) = \frac{k^4}{c}N(k) = \frac{1}{c_\beta c'_\theta}\left[1 + \left(\frac{k_s}{k}\right)^{\frac{1}{2}}\right]^{-1}, \qquad (2)$$

where c is the wave phase speed and c_β and c'_θ are coefficients determined from the form of the wave growth rate. The sheltering wavenumber, k_s, represents the wavenumber at which the local turbulent stress begins to be affected by sheltering by the longer wavelength waves. At low wavenumbers $k \ll k_s$, $B(k)$ is proportional to $k^{1/2}$ as in Phillips [1], but $B(k)$ becomes constant at high wavenumbers $k \gg k_s$.
The theory of Hara and Belcher [2] assumes that the wave induced stress is entirely due to nonbreaking waves, that is, the airflow is always attached to the wavy water surface. However, at high winds a significant portion of waves breaks and causes airflow separation and enhancement of the wave drag [3]. The objective of this paper is to examine the effect of breaking waves on the form of the equilibrium surface wave spectra.

THEORY

Momentum conservation in wave boundary layer

We begin by assuming that the total wind stress is a sum of the turbulent stress, the (nonbreaking) wave induced stress, and the breaking wave induced stress;

$$\tau_{tot} = \tau_t(z) + \tau_w(z) + \tau_b(z), \qquad (3)$$

where z is the height above the water surface. As in Hara and Belcher [1] the wave induced stress $\tau_w(z)$ is obtained by integrating the momentum input to waves in all angles and up to a wavenumber $k = \delta/z$, where δ is the normalized depth of the inner region [4]. The local turbulent stress $\tau_t^l(k)$, which forces waves at a wavenumber k, is then set equal to $\tau_t(z = \delta/k)$, that is, the turbulent stress evaluated at the top of the inner region.

It is assumed that the wave drag (momentum flux) due to a single breaking wave of a wavenumber k is determined by the length of its breaking crest and the wind speed evaluated at $z = \delta'/k$ (height that is equal to a set fraction of the wavelength) relative to the wave phase speed. The breaking statistic Λ is defined such that $\Lambda(\vec{k})d\vec{k}$ is the total length of breaking wave crests with wavenumbers between $\vec{k}$ and $\vec{k} + d\vec{k}$ per unit surface area. The breaking wave induced stress $\tau_b(z)$ is obtained by integrating the momentum input to breaking wave crests in all angles and up to a wavenumber $k = \delta'/z$.

Energy conservation in wave boundary layer

Since the braking wave induced stress depends on the wind speed rather than the local turbulent stress, it is necessary to obtain the vertical wind profile $u(z)$ together with the equilibrium wave spectrum. Following Hara and Belcher [5], the wind profile is determined from the energy conservation in the wave boundary layer:

$$\tau_{tot}\frac{du}{dz} + \frac{d\Pi}{dz} - \rho_a\varepsilon = 0. \tag{4}$$

Here, the vertical energy flux Π due to wave induced motion is set equal to a sum of the energy flux into nonbreaking surface waves integrated up to the wavenumber $k < \delta/z$ and the energy flux into breaking waves integrated up to the wavenumber $k < \delta'/z$. The viscous dissipation rate $\rho_a\varepsilon$ of the turbulent kinetic energy is parameterized in terms of the local turbulent stress following the approach used in one-equation models of turbulence.

Conservation of wave action spectrum

Within the equilibrium range the sum of the three forcing terms in (1) vanishes. The wind input term S_w is expressed as a sum of the input into nonbreaking waves and the input into breaking waves. The dissipation term D is set proportional to the breaking statistic Λ following Phillips [1]. All three forcing terms are assumed to be proportional to the cube of the degree of saturation.

RESULTS

The combination of the above three conservation constraints yields a coupled integral equations for the mean wind speed u and the degree of saturation B. After normalization the equations contain three nondimensional parameters: the wave age, the sheltering wave age (this parameter was defined by Hara and Belcher [5] and is proportional to Phillips' [1] constant of the equilibrium spectra), and a single new parameter that determines the breaking effect. The numerical solutions are then obtained with realistic values of these parameters.

The results of fully developed (mature) spectra show that the effect of breaking is relatively weak. The breaking effect is confined in an intermediate wavenumber range since both very long and very short waves propagate faster than the wind speed at the corresponding heights and therefore do not experience enhancement of the wind input.

With growing seas the breaking effect may be enhanced because breaking waves near the spectral peak receive enhanced momentum and energy fluxes from wind. Consequently, the wind profile is also modified significantly and the Charnock coefficient (normalized equivalent surface roughness) may increase by a factor or 2 or more due to breaking.

This theoretical model of the equilibrium wave spectra may be combined with a numerical wave prediction model to investigate the drag coefficient over complex seas.

References

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