UNIDIRECTIONAL STEEP WAVES IN WAVE TANKS

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Summary A nonlinear focusing process in which a single unidirectional steep wave emerges from an initially wide amplitude- and frequency-modulated wave group at a predicted position in the laboratory wave tank is studied both theoretically and experimentally. The spatial version of the Zakharov equation was applied in the numerical simulations. Experiments were carried out in two facilities which have substantially different scales: in the Tel-Aviv University wave tank, which is 18 m long, and in the Large Wave Channel in Hanover, which is 330 m long. Comparison between the experimental studies in both facilities and the corresponding numerical results is carried out. Good agreement was obtained between experiments and calculations. The effect of dissipation and bound waves is discussed.

Introduction

An ability to produce a single steep wave at a prescribed location in a laboratory wave tank of constant depth is often required for model testing in coastal and ocean engineering. Such waves can be generated by a superposition of a large number of waves at a given location and instant. This implies that effective focusing requires that the spectrum of the initial wave group generated at the wavemaker is wide. Dispersive properties of deep or intermediate-depth surface gravity waves can be utilized for this purpose. Since longer gravity waves propagate faster, a wave group generated at the wave maker in which wave length increases from the front to the tail may be designed to focus the wave energy at a desired location. Such a wave sequence can be seen as a group that is modulated both in amplitude and in frequency. The essentially nonlinear behavior of wave groups with high maximum wave steepness has been demonstrated in a number of studies. Due to nonlinear interactions, considerable variation of the wave spectrum along the tank is observed. The width of the spectrum in the current study precludes application of theoretical models that have severe limitations on the spectral width, like the cubic Schrödinger equation or its modification due to Dysthe (1979). An alternative theoretical model that is free of bandwidth constraints is the Zakharov (1968) equation. Unidirectional spatial version of this equation was derived and has been applied successfully in Shemer et al. (2001) to describe the evolution of nonlinear wave groups in the tank. The Zakharov equation is based on Hamiltonian formulation of the water-wave problem and does not account for non-conservative effects. The limits of applicability of the model equation are studied with respect to the problem of generation of a single steep unidirectional water wave. An experimental study of propagation of steep wave groups with wide spectrum is carried out in two laboratory wave tanks of different sizes. The experimental study is accompanied by numerical simulations based on the spatial version of the Zakharov equation.

Theory

The purpose of the present study is to obtain at a prescribed distance from the wave maker, $x = x_0$, a steep unidirectional wave group with a narrow, Gaussian-shaped envelope with the surface elevation variation in time, *?(t)*, given by

$$z(t) = z_o \, exp - (t / mT_0)^2 \, cos(w_o t), \tag{1}$$

where $w_0 = 2p/T_0$ is the carrier wave frequency, $?_0$ is the maximum wave amplitude in the group. The small parameter representing the magnitude of nonlinearity e is the maximum wave steepness $e = ?_0 k_0$. The parameter m determines the width of the group; higher values of m correspond to wider groups and consequently narrower spectra. The spectrum of the surface elevation given by (1) is also Gaussian. The wave field at earlier locations, $x < x_0$ is obtained from the computed complex surface elevation frequency spectrum at this location. To this end, the unidirectional discrete spatial version of the Zakharov equation is used:

$$i \cdot c_g \frac{\P B_j(x)}{\P x} = \sum_{w_j + w_l = w_m + w_n} V(w_j, w_l, w_m, w_n) B_l^* B_m B_n \, e^{-i(k_j + k_l - k_m - k_n)x}, \tag{2}$$

where c_g is the group velocity, V is the interaction coefficient and * denotes complex conjugate. This equation describes the slow evolution along the tank of every free spectral component $B_j = B(w_j)$ of the surface elevation spectrum in inviscid fluid of constant (infinite or finite) depth and accounts for Class I, or quartet, nonlinear interactions among various components. The dependent variables $B_j(x)$ representing the free components in the wave field, are related to the generalized complex 'amplitudes' $b_j = b(w_j, x)$ that are composed of the Fourier transforms of the surface elevation $\hat{z}(w_j, x)$ and of the velocity potential at the free surface $\hat{f}^s(w_j, x)$. The amplitudes b_j represent a sum of the free and the bound waves. The bound higher order components can be computed at each location once the free wave solution $B_j(x)$ is known. The spectrum corresponding to (1) is integrated using (2) from the planned focusing location x_0 backwards up to the wavemaker at $x = 0$. The waveforms derived from the computed spectra serve as the wavemaker driving signals, with corrections that account for the actual response of the wavemakers of both facilities.

Experimental Facilities and Procedure

The first facility is the Large Wave Channel (GWK) in Hanover, which is 330 m long, 5 m wide and 7 m deep. About 25 wave gauges are installed along the tank. The gauges are concentrated at the distance of 120 m from the wavemaker, where the steep wave emerges. In parallel, a similar study is carried out in the Tel-Aviv University wave tank which is

18 m long, 1.2 m wide and 0.9 m deep. In these experiments, the steep wave is tuned to appear at distances ranging from 5 m to 8 m from the wavemaker. Up to four wave gauges are used simultaneously. The gauges are placed on a carriage that can be moved along the tank. In both wave tanks wide spectrum short wave groups given by (1) with *m*=0.6 are studied. Numerical simulations are performed for 100 free modes and various amplitudes and carrier wave periods

Results

Am example of the measured and the computed variation of the spectral shape along the tank is given in Fig. 1 for the Hanover experimental facility. Corresponding variation of the surface elevation along the tank is presented in Fig. 2. Good agreement between the measurements and computations is clearly seen in these Figures. A single steep wave indeed appears in the tank at the planned location x_0 = 120 m, Fig. 2b. The effect of bound waves can be noticed in Fig. 1b where low and high harmonics in the amplitude spectra arise.

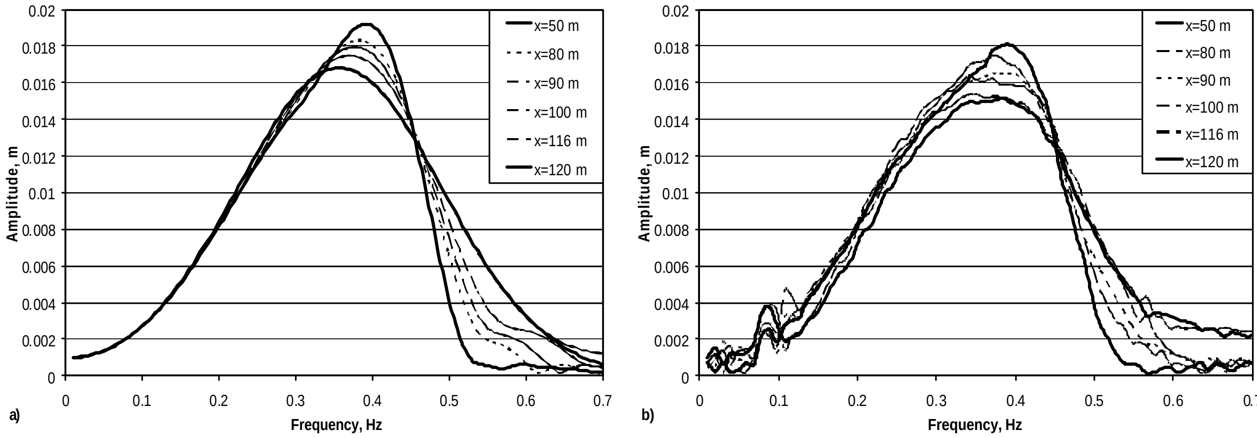

Figure 1. Amplitude spectra of wave group evolution for T_0=2.8 s, *m*=0.6 and the maximum wave steepness at the focusing location *x*=120 m, *e*=0.3: a) computation b) Hanover experiment.

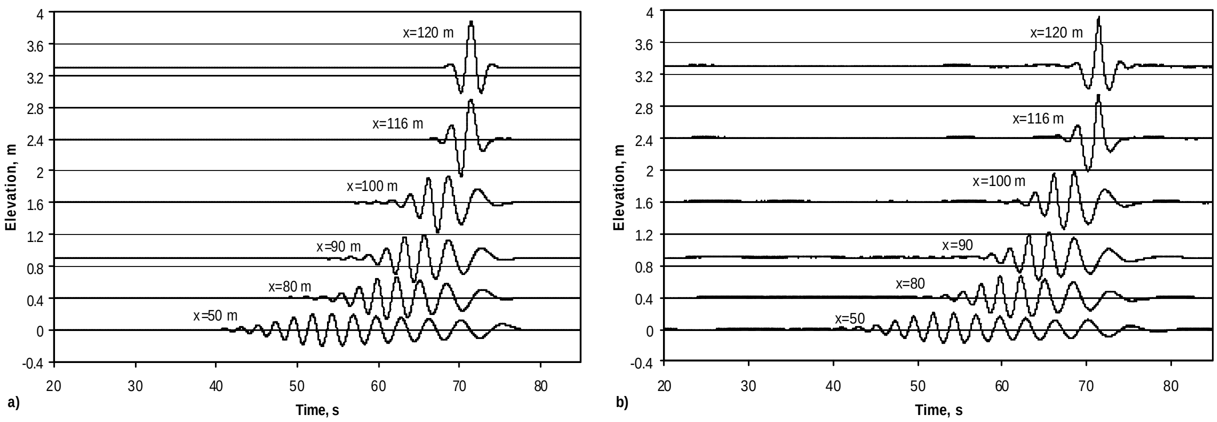

Figure 2. Wave elevation of wave group propagating along the large wave tank for T_0=2.8 s, *m*=0.6 and the maximum steepness at the focusing location *x*=120 m, ε=0.3: a) computation b) Hanover experiment.

The advantages and the limitations of application of the spatial Zakharov nonlinear water-wave model for simulations of wave field evolution in deep or intermediate-depth water are discussed based on the experiments carried out in both facilities. In particular, effects of evanescent modes, bound waves and dissipation are considered.

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