ON LOCAL VORTEX IDENTIFICATION

Pinaki Chakraborty, S. Balachandar, Ronald J. Adrian
Department of Theoretical & Applied Mechanics, University of Illinois at Urbana-Champaign, Urbana, IL, USA.

We explore the question of identifying vortical filaments in flow using a local criterion with two kinematic parameters. Based on the kinematic properties the vortex filament is required to satisfy the following requirements:

- The local flow in the frame of reference translating with the vortex should be swirling.
- The identified core geometry should be Galilean invariant.
- The separation between the swirling material points inside the vortex core remains small in the plane of the vortex, i.e. orbits of the material points are compact.

We follow the critical point approach [1] by adopting a frame of reference translating with the local fluid velocity. In this reference frame, the variation in the local velocity to the linear order is captured by the velocity gradient tensor $\boldsymbol{\nabla v}$. Using a similarity transformation it can be shown that in a local curvilinear coordinates spanned by the eigenvectors of $\boldsymbol{\nabla v}$ the local behavior of streamlines is governed by the eigenvalues of $\boldsymbol{\nabla v}$ [2].
The local swirling rate is determined by the imaginary part of the complex conjugate eigenvalue i.e. λ_{ci}. The time period for one complete revolution of the projected motion of the pathline in the plane spanned by the complex conjugate eigenvectors is given by $2\pi/\lambda_{ci}$. The second parameter is given by the ratio of the real to imaginary part of the complex conjugate eigenvalue and it measures the orbital compactness. In other words, in the plane of the vortex spanned by the complex conjugate eigenvectors the ratio of final to initial radial location of the spiraling path after one complete revolution increases as $exp(2\pi\lambda_{cr}/\lambda_{ci})$. In this regard, we identify $\lambda_{cr}/\lambda_{ci}$ to be the inverse spiraling compactness. Hence, these two parameters have unambiguous mathematical interpretation in terms of the local kinematics of the flow. These two kinematic parameters form the basis of our local vortex identification criteria. We identify the vortical region to be points in the flow that satisfy the dual requirement: $\lambda_{ci} > \epsilon$ and $\lambda_{cr}/\lambda_{ci} < \delta$, where ϵ and δ are positive thresholds and are determined from physical considerations of the problem.
We explore the relationships between these parameters and other commonly used local vortex identification methodologies, namely Q, Δ and λ_2 criteria. For the case of Q and Δ we get the following closed form relations:

$$Q = \lambda_{ci}^2 \left(1 - 3\left(\frac{\lambda_{cr}}{\lambda_{ci}}\right)^2\right) \tag{1a}$$

$$\Delta = \frac{\lambda_{ci}^6}{27}\left[1 + 9\left(\frac{\lambda_{cr}}{\lambda_{ci}}\right)^2\right]^2 . \tag{1b}$$

A significant aspect of the present approach is that regions of inward and outward swirling motion can be distinguished through the sign of $\lambda_{cr}/\lambda_{ci}$. For incompressible flows, inward swirling translates to stretching along the axial direction, hence the vortex can be interpreted as being in the process of intensification. Large positive values for the ratio $\lambda_{cr}/\lambda_{ci}$ above a set threshold corresponrs to a rapid outward spiral and thus will not qualify as a vortex filament.
It is clear from the above equations that $\Delta = 0$ criterion corresponds to $\lambda_{ci} = 0$ and similarly $Q = 0$ criterion corresponds to $\lambda_{cr}/\lambda_{ci} = 1/\sqrt{3}$. No similar simple relation exists between the $\lambda_2 = 0$ criterion and the rest. Here we interpret the $\lambda_2 = 0$ criterion in terms of λ_{ci} and the ratio $\lambda_{cr}/\lambda_{ci}$ for all possible configurations of the local velocity gradient tensor, $\boldsymbol{\nabla v}$. The key result is that $\lambda_2 = 0$ corresponds to a small range of value for the ratio $\lambda_{cr}/\lambda_{ci}$. Furthermore, except for a very small window in the configuration space, $\lambda_{ci} = 0$ criterion corresponds to the largest vortex core.
The above different criteria have been used in the past for a variety of flow fields. In some applications the vortical structures obtained by the different criteria have been observed to be remarkably similar [3], while in other applications significant differences between the different criteria have been highlighted [4]. Resolving such apparent contradiction has been an objective of the present study. Toward this end, we investigate the relationship between the different criteria at finite amplitude thresholds. The ease of local kinematic interpretation of the present criterion makes the choice of thresholds to be based on physical considerations. The time scale of the vortex filaments to be identified determines the value of ϵ and the orbital compactness of the desired vortex filaments determines δ. Here we explore the relation between the different criteria in the context of five different flows: Laminar Burger's vortex, swirling jet, isotropic turbulence, uniform flow over a sphere at moderate Re and turbulent channel flow. From these examples it becomes clear that in case of a strong vortex core, corresponding to a large positive threshold, the differences in the vortex core structure extracted by the different criteria is minimal. Significant differences emerge only in case of identifying weakly spiraling regions corresponding to small values of λ_{ci}.
For locating the intense vortices, we address the question: given the thresholds for λ_{ci} and the ratio $\lambda_{cr}/\lambda_{ci}$, what are the thresholds for the different criteria so that the vortical regions educed are comparable? Based on simple analysis we

propose the following equivalent threshold for the Q, Δ and λ_2 criteria, for extracting nearly identical intense vortical structures from the different criteria:

$$Q \geq Q_{th} = \epsilon^2 \tag{2a}$$

$$\Delta \geq \Delta_{th} = \frac{1}{27}\epsilon^6 \tag{2b}$$

$$\lambda_2 \leq (\lambda_2)_{th} = -\epsilon^2. \tag{2c}$$

We observe that for the case of Burger's vortex the difference in the size of the vortex core, as extracted by the different criteria, is significant at lower Re. With increasing Reynolds number, however, the differences between the different criteria rapidly decrease. In the case of isotropic turbulence, the well accepted worm-like intense vortical structures extracted by the different criteria with the equivalent threshold are nearly identical. The only exception is the Δ criterion, whose vortical structure includes additional "debris" as compared to the other criteria. We observe that these debris have high values of $\lambda_{cr}/\lambda_{ci}$, hence do not qualify as vortices. As can be observed from (1b) large values of the ratio $\lambda_{cr}/\lambda_{ci}$ also contributes to large values of Δ and thus erroneously misconstrued as intense vortical regions. Similar results were also observed for the case of wake flow over a sphere and the turbulence channel flow, with nearly identical vortex structures extracted by the different criteria using the equivalent threshold (except for the Δ criterion which also includes additional vortical debris).

We explore the question of interacting vortices with the model problem of a vortex ring. The strength and radius of a vortex ring are varied and the resulting vortex structure is educed using our identification criteria. For small radius and large strength, the vortex is subjected to self induced strain. Using our two parameters for identification of vortices, we explore the resolution of the criteria, i.e. its ability to distinguish the interacting vortex cores. This identification is compared with the other local schemes.

References

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