VISCOUS VERTICAL LENGTH SCALE SELECTION IN STRATIFIED FLUIDS

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Summary The evolution of pancake dipoles of different aspect ratios is studied in a stratified tank experiment. When the vertical Reynolds number is large enough, the vertical size of the dipole is shown to decrease whereas its horizontal circulation is conserved. This decrease of the dipole thickness is due to the peeling off of two boundary layers, on top and bottom of the dipole, where the fluid is slowed down by viscosity. The thickness of the dipole diminishes until a viscous length scale δ, defined by the vertical size of these boundary layers, is reached. Viscosity is therefore responsible for a fast vertical decorrelation of the flow. The mechanism that we evidence may play a significant role in the determination of the vertical length scale in strongly stratified flows.

GEOPHYSICAL CONTEXT

The emergence and evolution of pancake vortices in strongly stratified fluids have been studied intensely recently because of their role in the dynamics of geophysical flows. Of particular interest is the vertical length scale selection that constrains the energy and momentum exchanges determining for instance the turbulence spectra observed in the atmosphere and in the ocean.

EXPERIMENTAL SETUP AND OBSERVATIONS

The experiments are carried out in a tank of 1m x 2m base and 0.6m height filled with salt-stratified water. The dipole is generated by a pair of parallel vertical flaps as described in [1]. The experimental setup differs from [1] by the fact that the initial dipole is partially blocked by a vertical screen perpendicular to the moving direction of the dipole (figure 1). A single horizontal slice of the vortex column goes through the screen and forms a pancake dipole with an initial thickness that may be varied simply by adjusting the aperture on the screen.

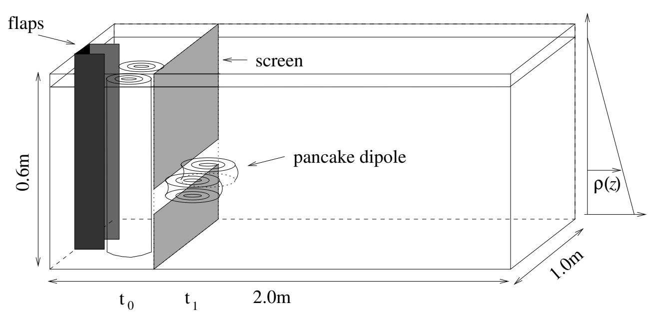


Figure 1. Experimental setup.

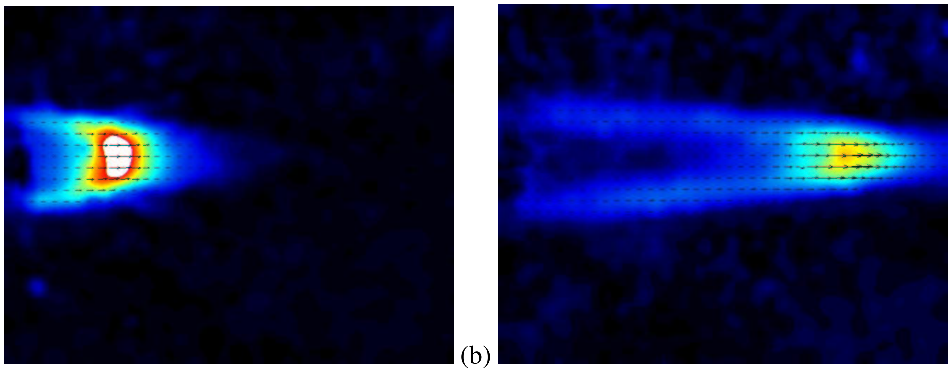
Three nondimensional control parameters can be defined for the dipole coming out of the screen: the Reynolds number $Re_0 = U_0 L_{h0}/\nu$, the horizontal Froude number $F_{h0} = U_0/NL_{h0}$ and the aspect ratio $\alpha_0 = L_{v0}/L_{h0}$, where U_0, L_{h0} and L_{v0} are, respectively, the initial translation speed and the horizontal and vertical length scales of the dipole, ν is the kinematic viscosity and N the Brunt-Väisälä frequency. Measurements of the velocity field were obtained by Particle Image Velocimetry (PIV).

In figure 2 we show two instants that depict the first stages of the evolution of a pancake dipole with initial control parameters $F_{h0} = 0.18$, $Re_0 = 182$ and $\alpha_0 = 1.27$. The velocity field in figure 2.b exhibits a horizontal V shape, the layers on top and bottom of the dipole being swept away generating two "wakes" in the upper and lower layers where the velocity is horizontal. A decrease on the vertical length scale is thus caused by the top and bottom layers that are slowed down and left behind. The present results contrast with the usual decay scenario for a dipolar structure in a stratified fluid, where the thickness of the layered flow slowly increases by diffusion while the circulation decays (e.g. [3]).

VISCOUS PEEL-OFF MODEL

We propose a model based on the low-Froude number theory first proposed by [4] where a scaling analysis of the equations for a stratified viscous fluid in the Boussinesq approximation gives at leading order the momentum equation in terms of the vertical vorticity

$$\frac{\partial \omega_z}{\partial t} + (\mathbf{u}_h \cdot \nabla_h)\omega_z = \frac{1}{\alpha^2 Re}\frac{\partial^2 \omega_z}{\partial z^2} \tag{1}$$



(a) (b)

Figure 2. PIV measurements on a vertical cross-section through the dipole symmetry axis of the velocity modulus $|V|$. The initial instant immediately after the dipole has come out of the screen is shown in (a). In (b) the same frame 90 seconds later.

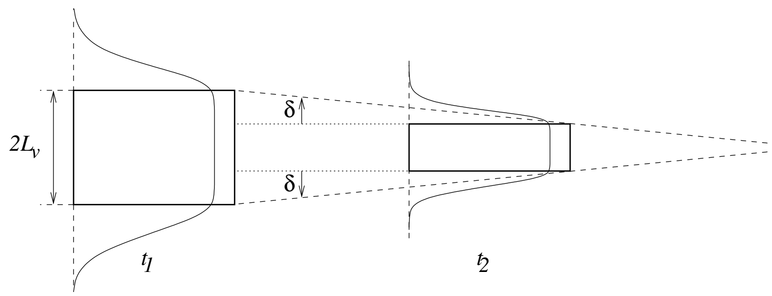


Figure 3. Schematic diagram of the two boundary layers during the slimdown regime.

where the subscript h denotes the horizontal field and α is the aspect ratio of the layer considered. In the boundary layer, the dominant balance principle imposes that the vertical length scale L_v should be such that $\alpha^2 Re = 1$ and the vertical scale

$$L_v = \delta \equiv L_h Re^{-1/2} \tag{2}$$

independent of N. In the core regions between the boundary layers $\alpha^2 Re$ is initially large and the motion may be assumed inviscid with a constant circulation. The height of the core region diminishes as the outer layers are slowed down —diffusing momentum to the still regions on top and bottom of the dipole— until it reaches the scale δ for which diffusion cannot be ignored. This scenario is shown schematically in figure 3. The two "boundary" layers of thickness δ on each side of the pancake dipole are left behind as the dipole moves until the core region vanishes. It should be noted that the mechanism described above relies on equation (1) which gives the leading order dynamics if $\delta \gg U/N$. This insures that vertical transport is negligible compared to diffusion. The other limit case when δ is smaller than U/N has been described by [2] and leads to a vertical scale proportional to the buoyancy scale $L_B = U/N$. The present theory completes this previous work in the case where viscous effect dominate over vertical transport.

CONCLUSIONS

The experimental observations on the present work give evidence of a new mechanism responsible for the formation of small-scale vertical structure in strongly stratified fluids. The creation of free "boundary" layers of thickness $\delta = L_h Re^{-1/2}$ observed in the experiments should occur when the ratio of the buoyancy length scale U/N to the viscous length scale δ is small, and whenever initially L_v is larger than δ, because there are no forces that counteract the viscous strain force. The present results manifest the need to consider viscous effects in any attempt of predicting the final vertical length scale of strongly stratified flows when the Reynolds number is lower than F_h^{-2} (i.e. when $\delta \gg U/N$).

References

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