STRONG CYCLONIC VORTICES OVER TOPOGRAPHY ON A BETA-PLANE

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Summary Strong cyclonic vortices over topography on a beta-plane were investigated by a joint analytical, experimental and numerical approach. The motion of the vortices shows complicated track deflections associated with meandering Rossby wave wakes. The track deflections can be further explored by a dynamic model which predicts similar results to those observed from the experiments, numerical calculations and some historical typhoon trajectories encountering the Island of Taiwan.

BASIC EQUATIONS AND THE DYNAMIC MODEL

A rotating *shallow water model* (SWM) is employed to simulate the behaviour of strong cyclonic vortices (Ro ~ O(1)-O(10)) over topography on a beta-plane. The bottom topographies considered in the present study include (i) an elliptical-shape topography, (ii) a gently sloping bottom and (iii) a paraboloidal free surface. The paraboloidal free surface is induced from the tank rotation, while the sloping bottom is implemented to represent a planetary beta effect. A gradient-wind balanced vortex model is adopted as the initial conditions which are the distributions of the surface depression and the azimuthal velocity. In order to take care of the viscous dissipation, an internal dissipation term is also incorporated in the momentum equations as suggested by Schär and Smith (1993).

The dynamic model of the strong vortices

According to the principle of conservation of potential vorticity (PV), and assume small variations of surface depression η^* and the bottom topography $h_B{}^*$, a non-dimensional PV conservation relationship for strong cyclonic vortices can be written

$$\frac{D}{Dt}\left(\beta_0 y + \beta_B h_B - \beta_v \eta + \varsigma\left(1 + Ro\left(\beta_B h_B - \beta_v \eta\right)\right)\right) = 0, \qquad (1)$$

where the non-dimensional beta parameters β_0, β_B and β_v, respectively, account for the planetary beta effect, elliptical-shape topography and the vortex depression. The proposed dynamic model predicts that at any instant, the velocity of the vortex

$$\vec{V} = \vec{V_c} + \vec{V_y} + \vec{V_B} \qquad (2)$$

consists of three contributions: $\vec{V_c}$, the component without planetary beta and topographic effect, and two other components $\vec{V_y}$ and $\vec{V_B}$ as explained below. During a small time interval, assume that the surface depression and the relative vorticity at the vortex centre are approximately unchanged. From Eq. (1), we can derive a *meridional adjusting velocity (MAV)* $\vec{V_y}$ of the vortex moving across the isobaths

$$\vec{V_y} = -\alpha\left(\frac{dh_B}{dt}\right)\vec{e_y} \text{ along the direction of } \vec{V_c}, \qquad (3)$$

where the topography-adjusting factor α is defined to be $\alpha \equiv \left(1 + Ro\varsigma_c\right)\beta_B / \beta_y$, where ς_c is the relative vorticity at the vortex centre. In the meanwhile, another *topography-adjusting velocity (TAV)* $\vec{V_B}$ is also induced as the vortex moving across the isobaths of the topography,

$$\vec{V_B} = \frac{1}{\left|\nabla h_B\right|}\left(\frac{dh_B}{dt}\right)\vec{e}_B \text{ along the direction of } \vec{V_c}, \qquad (4)$$

where $\vec{e}_B$ represents the non-dimensional unit vector along the topographic gradient.

EXPERIMENTAL SETUP

The experiments were performed in a rectangular glass water tank of horizontal dimensions 135 cm × 135 cm and 40 cm in depth rotating with a slow angular speed varying from 0.3 to 10 rpm. The generation of a cyclonic vortex was conducted by a 4-cm-diameter solid cylinder rotating in parallel to the axis of the rotating table. In the present experiment, the slope of the bottom was approximately 0.0538. Flow visualizations were performed by the horizontal streak photography and to measure the local surface depression by a vertical laser light sheet. A light sheet was generated from a 4-Watt Argon-ion laser, and the laser beam was reflected by an 8-facet rotating prism onto a thin light sheet of a fan region. The vertical light sheet was generated by a diode laser and was mounted directly on the tank. With careful manipulation on the position of the light sheet on the passage of the vortex, the surface profiles were recorded slice by slice during the passage of the vortex. Then, the actual vortex depression can be obtained by subtracting the corresponding profile with a reference frame. A particle track velocimetry (PTV) technique was also implemented to extract the horizontal distribution of velocity of the vortex.

NUMERICAL CALCULATIONS

The numerical method used in the present study closely related to Helfrich et al. (1999). The rotating shallow water model was numerically integrated using a public-domain, finite volume code CLAWPACK by LeVeque (1998). The main feature of CLAWPACK is a Godnov-type finite volume method in which the Riemann problems are solved at cell interfaces to properly resolve the wave structure. The SWM calculations were performed with a grid of size 320×320. All the calculations were performed on a Cray J916 machine at a non-dimensional time step 0.016 and approximately 11.6 CPU seconds were required for a single time step.

RESULTS AND DISCUSSION

Figure 1 shows the experimental streaklines photographs. Figures 2(a)-(d) shows a sequence of vorticity contours of the numerical simulation for a typhoon-like vortex on a beta-plane impinging upon a mountain with 2500 m height. The vortex moves to northwest due to the topographic beta effect and sheds secondary wakes at the downstream. Fig. 2(e) showes the comparisons of the vortex tracks obtained from the shallow water model (circles) and the proposed dynamic model (solid lines); the results are in close agreement with each other. Figure 3(a) showes some historical typhoons passing over Taiwan as reported by Wang (1980). Fig. 3(b) shows that qualitatively similar tracks were observed from the dynamic model calculations. From the time series of the vortex speed as shown in Fig. 3(c), we discover an unsteady motion of *deceleration-acceleration-deceleration-acceleration* (D-A-D-A) alternative type of motion.

CONCLUDING REMARKS

In summary, this study proposes a dynamic model which predicts the vortex speed under the influence of the planetary beta and topographic effect. The velocity $\vec{V}$ of the vortex in the proposed model consists of (i) the component without the planetary beta and topographic effect $\vec{V_c}$, (ii) the meridional adjusting velocity $\vec{V_y}$ and (iii) the topographic adjusting velocity $\vec{V_B}$. The results predicted by the model compare very well with experiments and numerical calculations based on the SWM. Further application of the proposed model to typhoon vortices encountering the island of Taiwan also shows very interesting comparison in the vortex trajectories.

References

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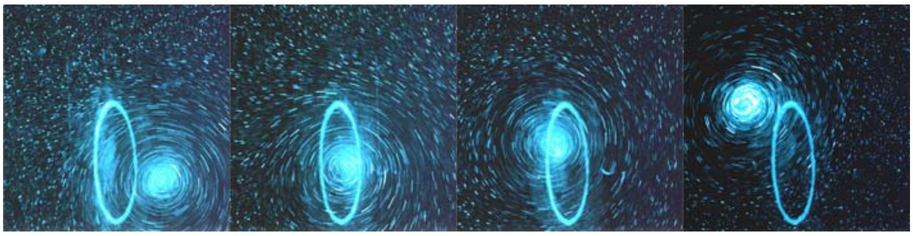

Figure 1 Experimental results of vortex streamlines evolutions.

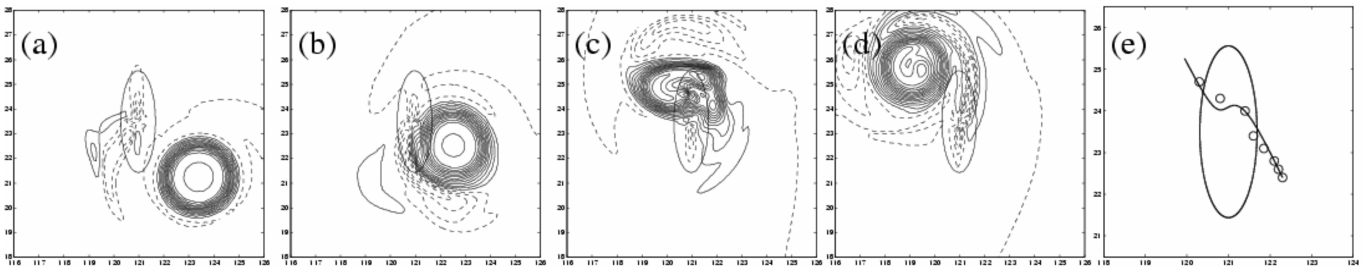

Figure 2 Comparison of shallow water simulation and the dynamic model results of vortex evolution over topography.

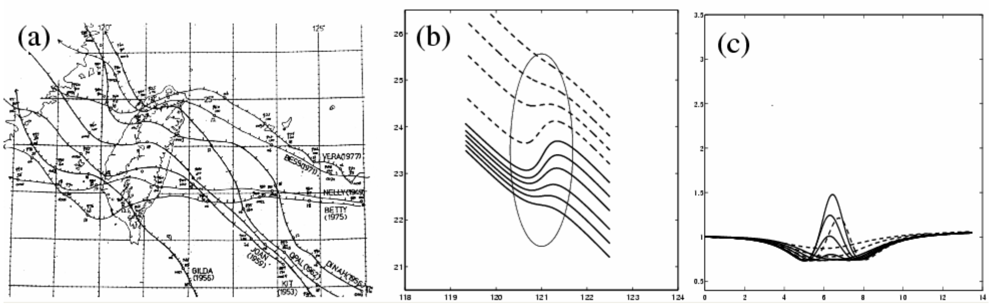

Figure 3 (a) Historical typhoon tracks, (b) vortex tracks calculated by the dynamic model and (c) time evolution of the drifting velocity of the vortex.