INSTABILITIES OF A VORTEX PAIR IN A STRATIFIED AND ROTATING FLUID

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Summary We present laboratory experiments on the three-dimensional instabilities developing on a counter-rotating vertical vortex pair in a stratified and rotating fluid. Four distinct types of instability have been identified on the cyclone or the anticyclone: the zigzag instability, the elliptic instability, the centrifugal instability and a new oscillatory instability. Numerical and theoretical stability analyses demonstrate that the latter oscillatory instability is a non-axisymmetric centrifugal instability.

INTRODUCTION

The three-dimensional stability of vortices in the atmosphere and the oceans is crucially controlled by the planetary rotation and the stable stratification. The rotation is indeed known to enhance both the centrifugal instability and the elliptic instability on anticyclonic vortices[1, 2]. Stratification effects are also responsible for the development of the zigzag instability[3].

Here, we present an experimental investigation of the three-dimensional instabilities developing on a counter-rotating vertical vortex pair in a stratified and rotating fluid. This work extends previous studies that have been conducted in the presence of either rotation [4] or stratification [3]. Four distinct types of instabilities have been observed on the cyclone or the anticyclone: the zigzag instability, the elliptic instability, the centrifugal instability and an oscillatory asymmetric instability. Numerical and theoretical stability analyses have been further conducted to determine the origin of the latter oscillatory instability which seems to have never been observed before.

EXPERIMENTAL SET-UP AND BASIC STATE

The vertical vortex pair is generated by computer-controlled flaps (i.e. by closing quickly two rotating vertical plates hinged to a vertical base) inside a 140cm$\times$140cm$\times$100cm tank mounted on a turntable which is 2.5m in diameter. The tank is filled with a stably stratified fluid with a working depth of 60cm.

PIV measurements have shown that the initial vortex pair can be well modeled by two Lamb-Oseen vortices separated by a fixed distance $b = 4$cm and with individual azimuthal velocity profile $v = \zeta a^2(1 - exp(-r^2/a^2))/2r$. The core radius a is always $a = 1$cm while the core vorticity ζ is controlled by varying the speed of closure of the flaps.

EXPERIMENTAL OBSERVATIONS

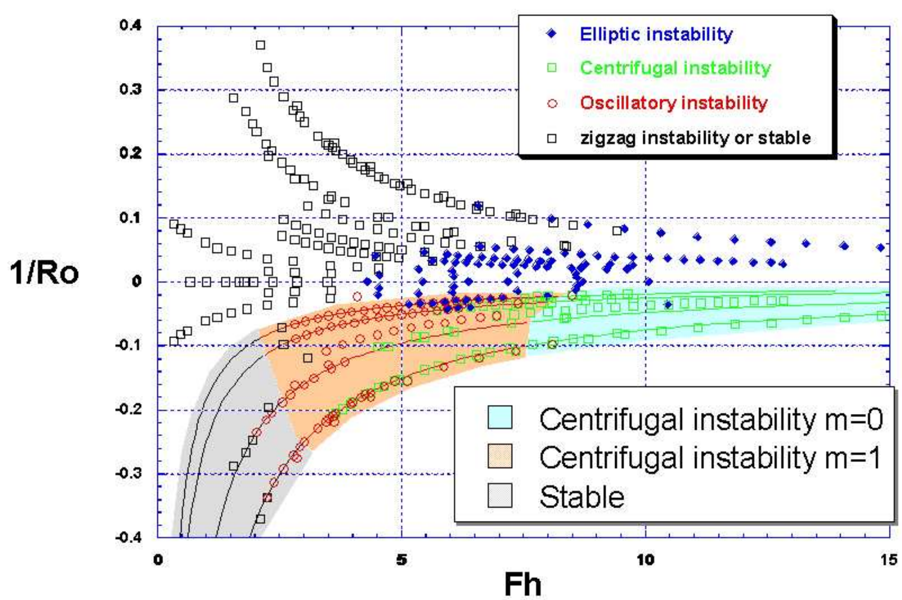
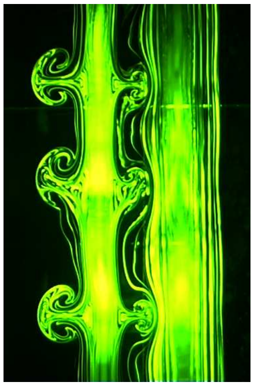
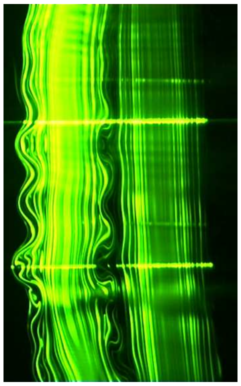


Figure 1. Experimental map of the instabilities in the parameter space $(F_h, 1/Ro)$. The symbols indicate the instability observed on the cyclone ($Ro > 0$) and the anticyclone ($Ro < 0$) for each experiment. The shaded regions show the results of the numerical stability analysis of the Lamb-Oseen vortex. See the text for more details.

The figure 1 shows the domains of existence of the different three-dimensional instabilities in the parameter space



(a)



(b)

Figure 2. Instabilities of the anticyclone (on the left of each picture): (a) Axisymmetric centrifugal instability, (b) Oscillatory instability ($m = 1$ centrifugal instability). The cyclone (on the right) is stable in both cases.

$(F_h, 1/Ro)$ where $F_h = \zeta/N$ is the horizontal Froude number and $Ro = \zeta/2\Omega$ is the Rossby number, N being the Brunt–Väisälä frequency and Ω the angular velocity of the rotating table. In this diagram, the instabilities observed in each experiment is reported by two symbols: for positive Rossby number, we indicate the instability observed on the cyclonic vortex, while for negative Rossby number the instability observed on the anticyclone is indicated.

For small rotation of the table (i.e. small $1/Ro$), we have observed the zigzag instability at small Froude number and the elliptic instability at large Froude number on both vortices as in [3]. As $1/|Ro|$ is increased, the elliptic instability develops with different wavelength and growth rate on the cyclone and the anticyclone. For larger $1/Ro$, the elliptic instability continues to be observed on the cyclone but tends to be stabilized by rotation effects for a given Froude number. In contrast, the anticyclone becomes subjected to two other types of instability: a centrifugal instability for large Froude number and an oscillatory instability for moderate Froude number. The centrifugal instability (figure 2(a)) produces axisymmetric toroidal vortices which are reminiscent of those of the Taylor-Couette instability. The oscillatory instability (figure 2(b)) is asymmetric and exhibits a standing oscillation. Horizontal cross-sections in the vortex pair have shown that the perturbation has an azimuthal wavenumber $m = 1$ that rotates in the same sense of rotation as the basic vortex.

NUMERICAL AND THEORETICAL STABILITY ANALYSIS

The observation of the centrifugal instability was predictable from the generalized Rayleigh criterion. In contrast, the oscillatory asymmetric instability is quite unexpected. To determine the physical origin of this new instability, the stability of an axisymmetric Lamb-Oseen vortex in a stratified and rotating viscous fluid has been investigated numerically for the same Reynolds number as in the experiments. In agreement with the generalized Rayleigh criterion, we have found that this vortex is centrifugally unstable for negative Rossby number: $Ro < -1$. However, the classical axisymmetric mode ($m = 0$) is the most unstable perturbation only for large Froude number : $F_h > 7$ (figure 1). In the range $3 < F_h < 7$, the most unstable centrifugal mode is the $m = 1$ azimuthal mode like the oscillatory instability observed in the experiments. Furthermore, as seen on figure 1, the domains of existence of the $m = 1$ centrifugal instability and the oscillatory instability are in very good agreement. The oscillatory instability is therefore of centrifugal nature. This conclusion is corroborated by the fact that the properties of the oscillatory instability are not in agreement with other possible candidates, namely the elliptic instability[2] or the Gent & McWilliams instability[5].

In order to explain further why a centrifugal mode with an azimuthal wavenumber $m = 1$ is selected for intermediate Froude numbers, we have conducted a WKB stability analysis for large vertical wavenumber. This approach leads to a generalization of the Rayleigh criterion to stratified rotating fluids and to non-axisymmetric perturbations. It clearly shows that the combination of stratification and viscous effects stabilize the $m = 0$ mode leaving room for the $m = 1$ mode to be the most unstable.

References

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