A new mixed nonlinear LES models for boundary layers

R.M. Kerr and P.G. Tucker
School of Engineering, University of Warwick, Coventry CV4 7AL UK

In 1970, Clark et al. [1] introduced a nonlinear model for the subgrid stress terms to complement linear dissipation such as the Smagorinsky eddy viscosity [13]. However, since then this simple formulation for adding more of the nonlinearity of the grid scales has been largely ignored by the turbulence modelling community. This is largely because it was not recognised that such nonlinear terms for a simple way to introduce backscatter effects, that is the energy from the subgrid scales that can appear in the resolved scales. This can be demonstrated by *a priori* analysis of interaction of filtered and sub-filtered scales in direct numerical simulations [6] Other approaches such as dynamically determining the coefficient in the Smagorinsky model C_s [11] and stochastical backscatter. [10] suffer from inherent problems in strong shears.

We will take the goal of LES research to be finding a formulation for the subgrid scales with the following properties:

- easy to implement
- dissipates approximately the correct energy,
- has roughly the correct transport properties,
- generates approximately the correct small-scale instabilities,
- and consistently does all of these for a variety of flow geometries.

This contribution will discuss comparisons between new large-eddy simulations (LES) and benchline flows (sophisticated LES and DNS) for two flows, primarily a pressure driven channel flow and secondarily decaying, isotropic turbulence. The channel flow is particularly challenging for LES because it has both a strong shear in the boundary layer and an interior that resembles isotropic turbulence. The classical LES approach [12] is to make the coefficient in the Smagorinsky eddy viscosity smaller in the boundary layer. The reason is that the value for homogeneous, isotropic turbulence, $C_s = 0.2$, creates too much dissipation in the boundary layer and suppresses the instabilities required to generate turbulence. The objective is either to effectively reduce C_s in the boundary or introduce fluctuations that could overcome the eddy-viscosity and drive the instabilities.

Figure 1 shows the profiles for several mixed non-linear models for the velocity profile in a classical, pressure-driven plane channel flow. The model that gave the best results is an existing LES model [14] plus a filtered nonlinear model based upon the Leray regularization of the Navier-Stokes equations [9]. The comparison with a sophisticated benchline LES [11] at $Re_\tau = 1050$, which in turn was based on comparisons with a high-resolution DNS [7], is nearly perfect, Direct comparisons with our own high-resolution DNS are in progress. Profiles of all components of the kinetic energy are also better than the competing mixed non-linear models [8, 15]. Coefficients for the eddy viscosity and nonlinear terms were determined by theoretical considerations of the energy cascade and discretization on the grid-scale, then checked by comparing an LES to a DNS of the energy decay and spectra for isotropic decaying turbulence,

The underlying LES channel model uses an eddy viscosity based on a URANS turbulent kinetic energy with a length scale damping function near the wall [15]. The (x,y,z) domain size of $2\pi \times 2 \times \pi$ is discretized on a 33 x 65 x 33 grid involving a cross-stream (y) geometric grid expansion factor of 1.15. This factor ensures y_+ values at 1st off-wall nodes are around unity. The grid is such that $Dx_+ \approx 200$ and $Dz_+ \approx 100$.

The decaying isotropic DNS is a higher Re version of a case [4, 5] that contains a strong initial transient. All the underlying decaying LES calculations were on a 64^3 mesh using the classical Smagorinsky eddy viscosity [13]. When a small physical viscosity is added for stability, the basic model agrees with the DNS decay rate as does the spectra of the LES until near the wavenumber cutoff, at which point there is a small anamolous at the high wavenumber end of the LES spectrum.

To each of these LES models, a filtered non-linear Leray model [3] is added. This filters non-linear advection terms and adds the following filtered non-linear term at the grid scales:

$$\frac{1}{1+\alpha^2 k^2}\text{FFT}\left\{\alpha^2(u_{i,k}u_{j,k} + u_{i,k}u_{k,j})\right\} \text{ with } \alpha^2 = \Delta^2. \text{ where } \Delta \text{ is the mesh.} \quad (1)$$

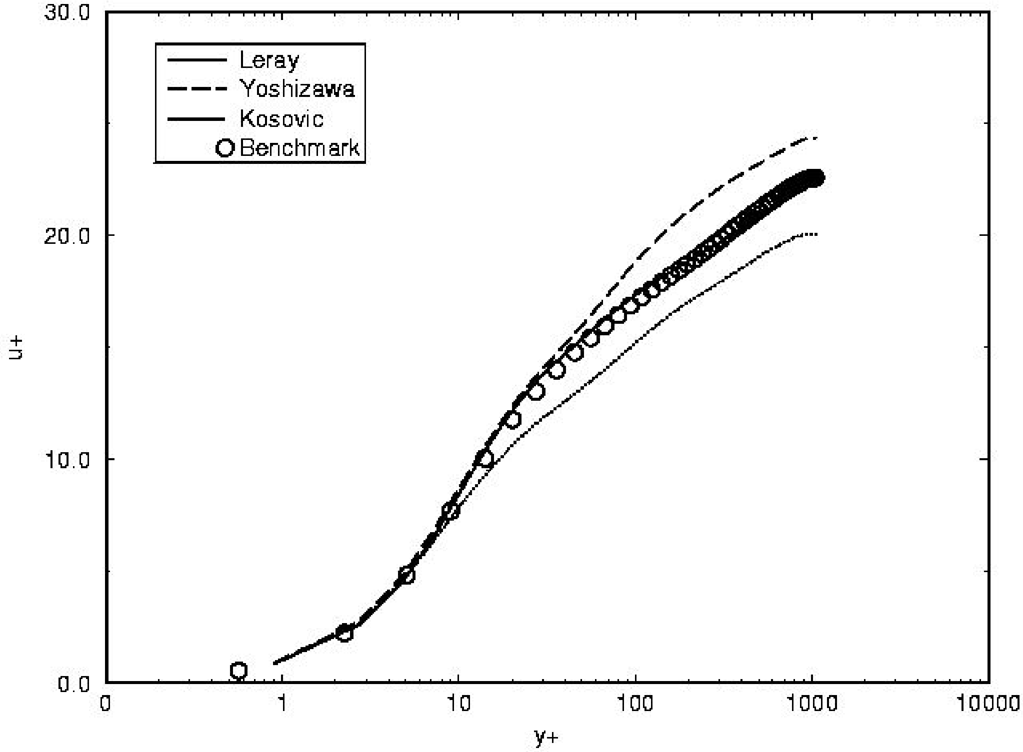

Figure 1: Mean profile for plane channel flow produced by several mixed nonlinear models [8, 15], including ours (Leray). Benchmark is dynamic LES [11]

The indicated Helmholtz filter is used with the decaying calculations and the channel code uses a multi-grid filter. This non-linear term is very similar to that proposed by Clark et al. [1], whose form reduces to the first term of (1) with a coefficient of $(1/12)\Delta^2$. Doing these operations succeeds in eliminating the anamolously high energies near the wavenumber cutoff in the pure Smagorinsky case. The non-linear terms of Clark et al. and LANS-α [2] were also tested, yielding worse results for the mean velocity profile.

References

[1] R.A.,Clark,J.H.,Ferziger,and W.C.,Reynolds J. Fluid Mech. **91**, 1 (1979). '

[2] C.,Foias,D.D.,Holm,and E.,Titi Physica D **152**, 505 (2001).

[3] B.J.,Geurts,and D.D.,Holm Phys. Fluids **15**, L13 (2003).

[4] J.R.,Herring,and R. M.,Kerr Phys. Fluids A**5**, 2792 (1993).

[5] D.D.,Holm,and R.M.,Kerr Phys. Rev. Lett. **88**, 244501 (2002).

[6] Kerr, R. M., J.A. Domaradzki, & G. Barbier 1996: Small-scale properties of nonlinear interactions and subgrid-scale energy transfer in isotropic turbulence. *Phys. Fluids* **8**, 197.

[7] J.,Kim,P.,Moin,and R.D.,Moser J. Fluid Mech. **177**, 133 (1987).

[8] B.,Kosović J. Fluid Mech. **336**, 151 (1997).

[9] J.,Leray Acta Mathematica **64**, 193 (1934).

[10] P.J.,Mason,and D.J.,Thomson J. Fluid Mech. **242**, 51 (1992).

[11] U.,Piomelli Phys. Fluids A**5**, 1484 (1993).

[12] U.,Schumann J. Comp. Phys. **18**, 376 (1975).

[13] J.,Smagorinsky Mon. Weath. Rev. **93**, 99 (1963).

[14] P.G,Tucker,and L.,Davidson Computers and Fluids **33**, 267 (2003).

[15] A.,Yoshizawa Phys. Fluids A**1**, 1293 (1989).