INTERACTION BETWEEN A COLUMNAR VORTEX AND EXTERNAL TURBULENCE

Naoya Takahashi, Takeshi Miyazaki
Department of Mechanical Engineering and Intelligent Systems, University of Electro-Communications, 182-8585, JAPAN

Summary The interaction between a columnar vortex and external turbulence is investigated numerically. As the columnar vortex, the Lamb-Oseen vortex and the q-vortex are used. The columnar vortex is immersed in an initially isotropic homogeneous turbulence field, which itself is produced by a direct numerical simulation of decaying turbulence. Using visualization techniques, we investigate the formation of inhomogeneous fine turbulent eddies around the columnar vortex, the vortex-core deformations and the dynamical evolution in the passive scalar field.

INTRODUCTION

Interactions between intense elongated vortices and surrounding turbulent motion often occur in engineering and environmental flows, and they occur naturally in most homogeneous and sheared turbulent flows. Several experimental and computational studies have been undertaken on the instability of vortices interacting with background turbulence. These interactions are thought to play an important role in the three-dimensionalization of the flow fields. It is also of practical interest to estimate the lifetime of trailing vortices under the influence of atmospheric turbulence. In this paper, we investigate the dynamical evolution of the vortex structure and the passive scalar structure, appeared due to the isotropic homogeneous turbulence interacting with the columnar vortex. The characteristics of the flow field are analyzed by using several flow-visualization techniques. The statistics of the turbulence around the columnar vortex are investigated by computing the two-point energy spectrum tensors, and they are compared with the results of a linear rapid distortion theory (RDT) [1]. The growth of the velocity and vorticity disturbances are proportional to $t^{0.9}$ and t^1 respectively. Although the results are qualitatively consistent with RDT, that do not agree quantitatively with the prediction of RDT($\propto t^2$).

NUMERICAL METHOD

We solve the Navier-Stokes equation for incompressible fluids and the transport equation for the passive scalar (such as a passive temperature field or a dye) under periodic boundary conditions with period 4π. A spectral method is used to solve the equations. The time integration is performed using the fourth order Runge-Kutta-Gill method. The simulations are done with resolutions of 256^3 and 512^3.

The columnar vortex is immersed in an initially isotropic homogeneous turbulence field, which itself is produced numerically by a direct numerical simulation of decaying turbulence.

The initial condition is set up with an energy spectrum of $k^4 \mathrm{e}^{-2k^2}$. Its Reynolds number is 500, based on the velocity as the root mean square of the field, and the length of the inverse of the wavenumber k at which the initial energy spectrum is maximal and based on the kinematic viscosity ν. The initial turbulent field is produced as decaying turbulence. The Reynolds number based on the Taylor micro scale is about 126. In these initial fields, a coherent fine scale structure of 'worms' is observed.

As the columnar vortex, we use the q-vortex, which is a model for trailing vortices. The q-vortex is defined by

$$(U_r, U_\theta, U_z) = \left(0, \frac{\Gamma_0}{2\pi r}\left\{1 - \exp\left(-\frac{r^2}{r_0^2}\right)\right\}, \frac{\Gamma_0}{2\pi r_0 q}\exp\left\{-\frac{r^2}{r_0^2}\right\}\right), \tag{1}$$

where U_r, U_θ and U_z are the radial, azimuthal and axial components of the velocity field, and q is the swirling parameter. In this paper, r_0, the initial radius of the columnar vortex, and $T = 2\pi r_0/(\Gamma_0/2\pi r_0)$, the time period for a fluid particle on the surface of the vortex core to make one revolution, are used as the characteristic length and time scales to render the results dimensionless. The initial circulation Γ_0 is an arbitrary parameter, so we set the circulation strong enough to dominate the vortex dynamics of the flow field as $\Gamma_0 = 40 r_0^2 \omega_{r.m.s.}$ where $\omega_{r.m.s.}$ is the root mean square of the vorticity of the initial turbulence. Then the Reynolds number of the columnar vortex Γ_0/ν becomes about $20,000$ to $80,000$. We consider three values of q, that is $-\infty$ (a.k.a. Lamb-Oseen vortex), -0.45 (unstable case) and -1.5 (marginally stable case).

RESULTS

The Lamb-Oseen vortex ($q = -\infty$)

In this case, the columnar vortex corresponds to the Lamb-Oseen vortex, which is an exact solution of the Navier-Stokes equation. The linear stable columnar (Lamb-Oseen) vortex undergoes a deformation due to the interaction, and the vortex wraps worms around its surface in a spiral structure (Fig. 1(a)). On the surface, external velocity disturbances are blocked by the vortex and they cannot penetrate into the vortex core directly ('blocking effect'), whereas various types of vortex waves (Kelvin waves) are excited. The disturbances in the axial direction were suppressed. This result is consistent with the Taylor-Proudman's theorem.

Scalar advection of the interaction ($q = -\infty$)

The initial scalar field is initialized with the profile $s_0(z)$, which depends only on the axial component of z and has a Gaussian profile. Figure 1(b) is an instantaneous snapshot of the scalar field with enstrophy. This figure shows noticeable advection of the passive scalar field around the surface of the vortex core. This phenomenon may be due to the fact that the blocking effect of the velocity field is excited where the mean axial gradient of the scalar field is large. Another characteristic advection is observed separated from the vortex core. The coherent fine scale structure (worm) wrapps the passive scalar; i.e., the only swirling motion of the vortical structure becomes important in the scalar advection.

Unstable case of q-vortex($q = -0.45$)

The q-vortex with $q = -0.45$ is most unstable against the bending disturbance with $m = 1$[2]. The vortex core tends to form helical structures with two blades. The structure corresponds to the linear instability. At about the time of the saturation of the linear instability, small vortex ring-like structures occur around the surface of the blades as shown in Fig. 2(a). After a short time, the columnar vortex disolves abruptly, which indicates that the small vortex rings may correspond to the secondary instability.

Less stable case of q-vortex ($q = -1.5$)

We investigate the less unstable case in which the q-vortex has the swirl parameter $q = -1.5$. This parameter is selected so that the q-vortex is almost neutrally stable, and the q-vortex with $|q| > \sqrt{2}$ is stable for disturbances (WKB modes) at the smallest wave numbers. Fig. 2(b) shows the isosurface of the vorticity. Inside the vortex core, the emergence of thin and strong spiral structures, that are different structures of worm, are observed. This indicates that the nonlinear interaction is concentrated inside the core. These structures generated around the surface of the core are wound up by the columnar vortex, which stretches and thins the structure.

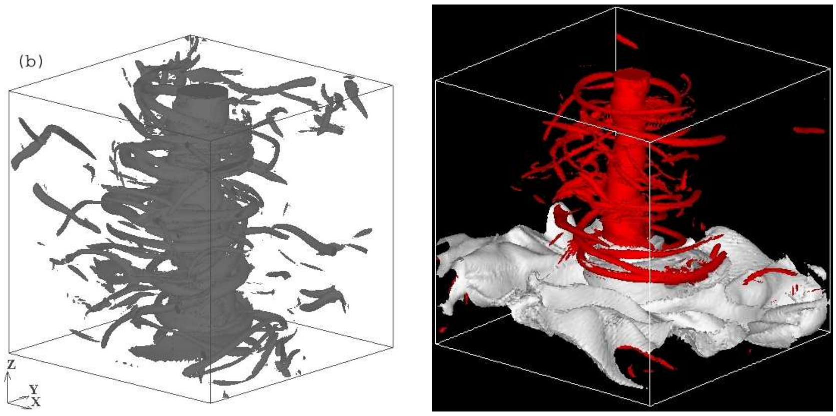

Fig. 1 (a) Iso-surfaces of vorticity. (b) Iso-surfaces of vorticity (red) and passive scalar (white). ($q = -\infty$, Lamb-Oseen vortex).

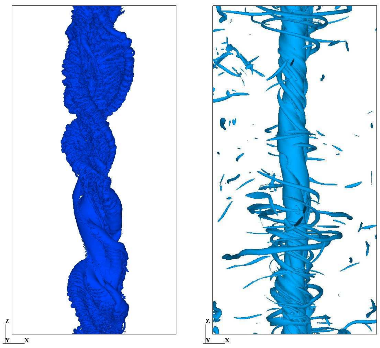

Fig. 2 (a)Iso-surfaces of vorticity (q-vortex, $q = -0.45$ (unstable case)). (b)Iso-surfaces of vorticity (q-vortex, $q = -1.5$ (less stable case)).

CONCLUSION

We investigated the interaction between a columnar vortex and external turbulence using direct numerical simulations.
The linear stable columnar (Lamb-Oseen vortex, $q = -\infty$) vortex underwent a deformation due to the interaction. The vortex wraps worms around itself to form spiral structures. Inside the vortex core, axisymmetric and bending vortex waves are excited. The interaction promotes scalar advection on the surface of the vortex core.
In the case of the unstable columnar vortex (q-vortex, $q = -0.45$), helical structures occur due to the linear instability. At the saturation of the linear instability, the secondary instability sets in and the columnar vortex disolves abruptly.
In the case of the marginally stable columnar vortex (q-vortex, $q = -1.5$), thin and strong spirals form inside the vortex core. These structures are stretched by the differential rotation around the columnar vortex, and they are dissipated gradually.

References

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