TOPOLOGICAL ASPECTS OF THE TORNADO PROBLEM

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Summary We examine the topological aspects of a concentrated vortex structure, a tornado, in inviscid fluid. A laboratory study is complemented by numerical simulations of vortex dynamics taking into account the topological structures of vortex knots. Natural observation performed in Kazakhstan (August 2002) agree reasonably well with laboratory experiments.

TORNADO VORTEX PROBLEMS

We have considered three-dimensional concentrated vortex problem related to the topological properties of intensive tornado structures.

Fig. 1

Tornado in Kazakhstan (August 2002)

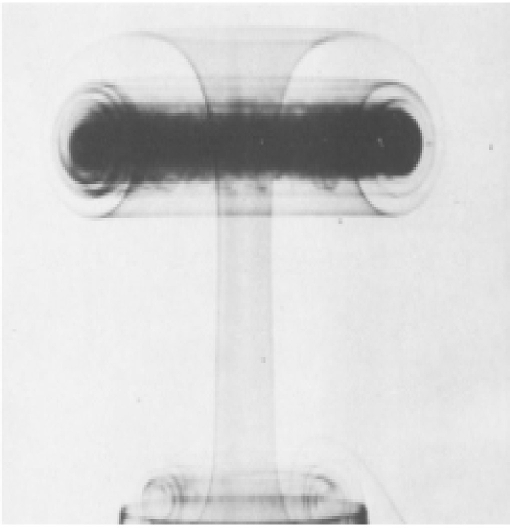
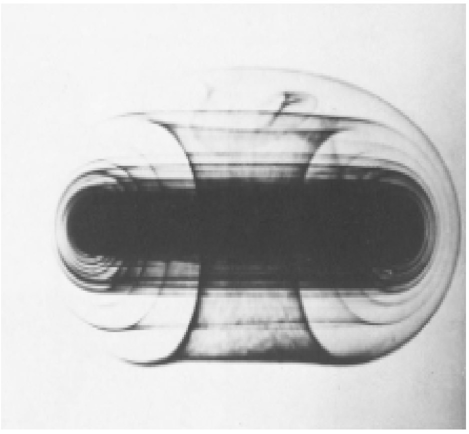
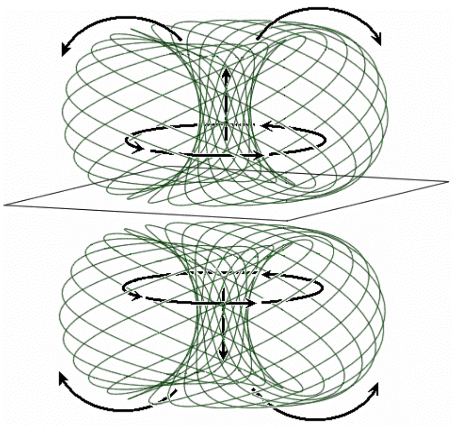


Fig. 2

Vortex ring experiments (Weyer)

It is assumed that initially the tornado can be modelled as a torus (two-dimensional closed manifold) over a rigid surface. The mirror image of the torus (Figure 1) provides the possibility to consider the development of this vortex structure in an unbounded domain.





The mathematical statement of the problem for the potential flow outside the vortex core is:

$$t \geq t_0: \qquad \Delta\phi = 0 \qquad \textbf{in} \quad D^+ \tag{1}$$

$$\frac{\partial\phi}{\partial n} = W_n \qquad \textbf{on} \quad L_d \tag{2}$$

$$\left(\frac{\partial\phi}{\partial n}\right)^+ = \left(\frac{\partial\phi}{\partial n}\right)^-, \quad p^+ = p^- \qquad \textbf{on} \quad L_v \tag{3}$$

$$\left|\nabla\phi\right| = V_\infty < \infty \qquad \textbf{at} \quad \infty \tag{4}$$

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 = p \tag{5}$$

$$t = t_0: \qquad L_0, \quad \phi^+\big|_{t=0} = \phi_0^+ \tag{6}$$

We have performed the experimental studies of the tornado-like structures using the idea suggested by Weyer (1889). The phenomenon of vortex hysteresis is experimentally observed.

We have presented a system of ordinary differential equations describing concentrated vortex dynamics that allow for efficient numerical simulation.

$$\frac{d\vec{r}_v}{dt} = \vec{V}(\vec{r}_v(t), t)$$

$$\vec{r}_v(t_0) = \vec{r}_{v0}$$

The numerical model [2] may be readily extended to describe more complex physical situation.

The Lyapunov stability conditions is essential while studying the Euler characteristic of the vorticity field. The possibility of effective stirring by a steady motion in the vortex core is pointed out. These problems might be benchmark ones for various numerical schemes in the vortex dynamics.

References

[1] Cherniy D. I.: Approximation of solution of initial boundary problem with moving boundaries. *J. Comput. Appl. Math* . 82 :112-123, 1997.

[2] Dovgiy S.A., Cherniy D. I.: About topology of the boundary of MDV. *IX Int. Symp. “Discrete Singularities Methods in Mathematical Physics-2000”*, OSU, Orel, 2000, 465-467.