

MIXING IS AN AGGREGATION PROCESS

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Summary Experiments show how a stirred scalar mixture relaxes towards uniformity through an aggregation process. The elementary bricks are stretched sheets whose rates of diffusive smoothing and coalescence build up the overall mixture concentration distribution. The cases studied in particular include mixtures in two and three dimensions, with different stirring protocols and Reynolds numbers which all lead to a unique family of concentration distributions stable by self-convolution, the signature of the aggregation mechanism from which they originate.

INTRODUCTION AND OBSERVATIONS

A mixture is a transient state between the initial segregation of the constituents, and their ultimate homogeneity. The overall mixing process of a drop of dyed fluid in a stirred medium involves two phenomena : a process of dispersion of the drop in the diluting medium by which the phases interpenetrate, and a process of interaction between the dispersed elements from which homogeneity arises. It is shown here that the mixture's evolution is directed by a random aggregation process essentially independent of the detailed stirring mechanism. In a first experiment (Fig.1a), a turbulent jet of water plus dye discharge in a square, transparent, long duct at $Re \simeq 10^4$. A second experiment consists in stirring a blob of dye with a rod in a thin layer of very viscous fluid at $Re \simeq 10^{-1}$, by a two-dimensional, quasi-periodic protocol (Fig. 1b).

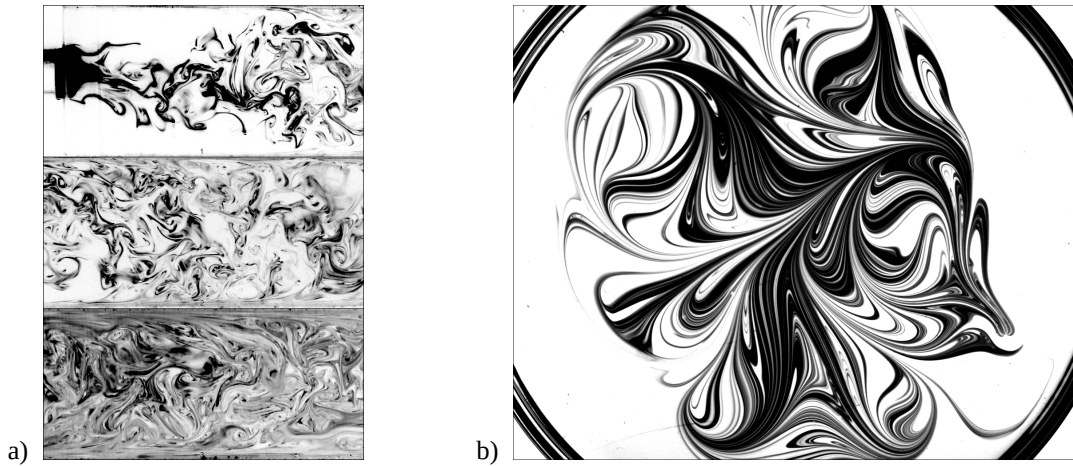


Figure 1. a) Successive mixing states of a dye discharging from a jet in a square duct. b) Mixture realized by stirring a dye in a viscous two-dimensional layer by a periodic protocol.

In both cases, the distribution $P(C)$ presents a skewed, bell shape which gets narrower around $\langle C \rangle$, very well described by a family of one parameter distributions, namely Gamma distributions

$$P(X = C/\langle C \rangle) = \frac{n^n}{\Gamma(n)} X^{n-1} e^{-nX}. \quad (1)$$

The fairness of the fit holds for the whole concentration range, down to quite low probability levels, and accounts for the deformation of $P(C)$ through the single parameter n which depends on the protocol: it increases like the power 5/2 of time (or downstream location) in the channel case, and like the power 3/2 of the number of stirring cycles in the two-dimensional case (Fig.2).

STRETCHING ENHANCED DIFFUSION AND MIXTURE COMPOSITION

Stirring motions progressively convert a compact blob in a set of sheets of increasing surface and decreasing thickness [1]. Let $s(t)$ be the distance between two material particles in the direction z perpendicular to a sheet, and $\sigma(t) = \partial \ln s(t) / \partial t$ its rate of compression. The concentration profile $c(z, t)$ across the sheet results from an equilibrium between stretching induced compression and diffusive spreading. The thickness reduction process goes on until $\sigma(t)$ is balanced by the rate of diffusive spreading $D/s(t)^2$. This defines the mixing time t_s of the sheet. If for instance the sheet thickness is $s(t) = s_0(1 + \gamma t)^{-\beta}$ where γ is an elongation rate, then $\gamma t_s \sim (\gamma s_0^2/D)^{\frac{1}{2\beta+1}}$.

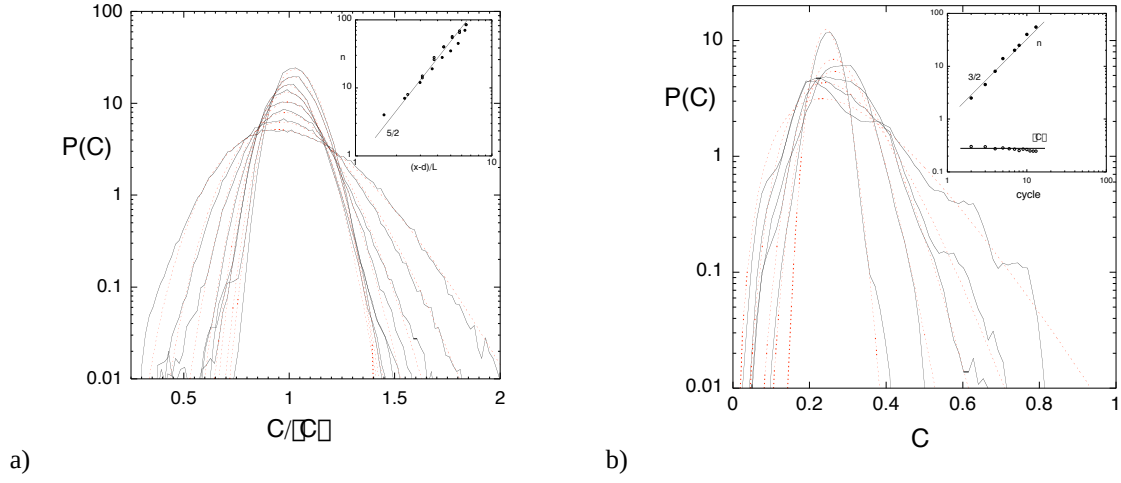


Figure 2. a) Downstream evolution of the concentration distribution $P(C)$ as the dye progresses along the duct as shown on Fig. 1 and fitting parameter n of the distributions of Eq. 1 as a function of the downstream distance $(x-d)/L$. $\circ$: $Re = 10^4$, $\bullet$: $Re = 5 \times 10^3$. b) Concentration distributions for the two-dimensional protocol as a function of the number of stirring cycles, and fitting parameter n . $Re \simeq 10^{-1}$.

Beyond this characteristic time, the maximal concentration in the sheet $c(0, t)$ which is such that $c(0, t)(Dt)^{1/2}/s(t)$ is, by mass conservation, constant, decays as $c(0, t) \sim (\gamma t)^{-\beta-1/2}$. In incompressible two-dimensional flows where the length of material lines grow like γt [2], the mean transverse thickness of the scalar sheet decrease as $s(t) = s_0/\sqrt{1 + (\gamma t)^2}$, providing $c(0, t) \sim (t/t_s)^{-3/2}$ for $t > t_s$, with $t_s \sim \frac{1}{\gamma} Pe^{1/3}$, where $Pe = \gamma s_0^2/D$ is a Péclet number. If material surfaces in three dimensions grow like $(\gamma t)^2$, then [3], $s(t) = s_0/(1 + (\gamma t)^2)$, providing $c(0, t) \sim (t/t_s)^{-5/2}$ for $t > t_s$, with $t_s \sim \frac{1}{\gamma} Pe^{1/5}$. These timescales are the relevant mixing times as soon as the inverse of the elongation rate γ^{-1} is smaller than the diffusive time of the sheet constructed on its initial size s_0^2/D , that is for $Pe \gg 1$.

However, the sheets interact as they move in the flow so that their diffusive boundaries interpenetrate to give rise to new sheets whose concentration profile is the *addition* of the original ones owing to the linearity of the diffusion process. Because of the irregular stirring motions, the addition of the concentration levels is made at random among those available in the population $P(C)$ at time t which therefore evolves by self-convolution as [4]

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial C} \left(\left\langle \frac{dc}{dt} \right\rangle P \right) + \frac{dn}{dt} \left(-P + P^{\otimes(1+1/n)} \right), \quad (2)$$

whose asymptotic solution is a Gamma distribution of order n . The piling-up of the concentration levels by coalescence contributes, through the second term of Eq. 2, to the increase of the average concentration by a factor given by $\exp\{\int dn/n\} = n$ and is balanced by the damping factor when $n = 1/c(0, t)$. Stretching motions in 3D form sheets whose maximal concentration $c(0, t)$ decays like $t^{-5/2}$ whereas 2D motions form filaments whose maximal concentration decays like $t^{-3/2}$. The coalescence mechanism giving rise to Eq. (1), will therefore make the average concentration $\langle C \rangle$ constant if

$$n \sim t^{-3/2} \quad \text{in 2D, and} \quad n \sim t^{-5/2} \quad \text{in 3D,} \quad (3)$$

as expected from Fig. 2. The mechanism building-up the concentration distribution $P(C)$ solely relies on *random additions* of concentration levels, independently of the *rate* at which these additions are made.

CONCLUSIONS

Composition fields obtained at a very low Reynolds number are very similar to those obtained at much larger Reynolds number. The reason is that the evolution mechanism is the same. The motion of the rod in the two-dimensional viscous fluid plays the role of the random stirring motions in higher Reynolds number flows. Their role is to ensure the independence –in the statistical sense of Eq. (2)– of the concentration levels additions giving rise to the self-convolutive construction of the mixture composition; a particularly simple paradigm for the impact of turbulence on mixing.

References

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