EVALUATION OF TRANSPORT PROPERTIES BY EXCHANGE MATRIX METHOD

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Summary This work is devoted to the study of transport properties of materials in chaotic two-dimensional stirring, using Spencer & Wiley matrix method. This study is important for applying in problems of pollutant transport (such as petroleum patches) in tidal flows. In order to construct this special exchange matrix we use an approximation of such flows suggested by Zimmerman, who adopted the idea of chaotic advection, first put forward by Aref. Then for a quantitative estimation of the transport properties we explore a coarse-grained density description introduced by Gibbs and Welander. Such coarse-grained representations over an investigation area show a "residence place" for the pollutant material at any instant. The exchange matrix can show transport of patches or particles from any place in the area under consideration to an arbitrary location in the tidal sea and time if it happens.
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INTRODUCTION

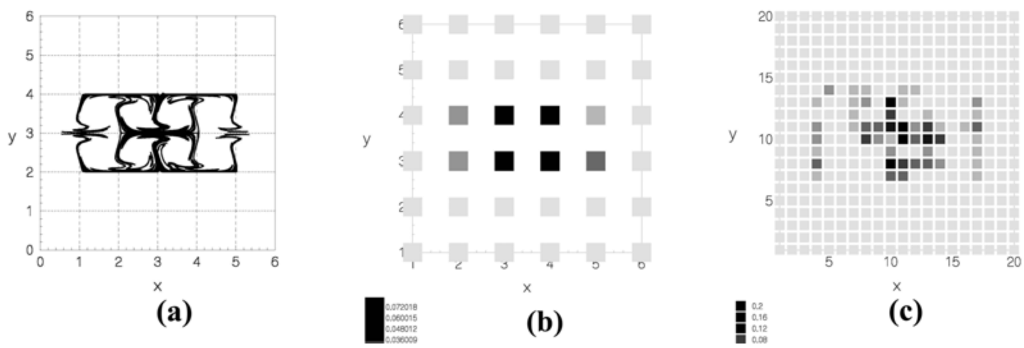
The study of transport of pollutants is an important issue. This work deals with exchange matrix method described by Spencer & Wiley [1] and in the paper [2]. We apply this method to problems of pollutant transport, for example, of petroleum patches in tidal flows. A good approximation of such flows was suggested by Zimmerman [3]. He deviated from earlier attempts at modeling tidal flows by means of turbulence theory and adopted the idea of chaotic advection, first put forward by Aref [4]. In this pioneering paper it was shown that chaotic mixing of a passive tracer (which in our case will be the petroleum patch) may occur even in deceptively simple flow systems. Zimmerman [3] realized that the macroscale flow in a tidal sea is not much more complicated than chaotic advection model and so should lead to similar phenomena. In particular, he suggested that the many fine striations often observed in scalar dispersal were due to chaotic motion driven by the macroscale, rather than microscale turbulent mixing. By numerical simulations he demonstrated that the patterns formed by dispersed passive particles were very similar to field observations. In the recently published tutorial paper Aref [5] described numerous applications of chaotic advection in fluid mechanics, in general, and in geophysical and geological flows, in particular. It was shown in many other works [6,7] how important it is to identify the parameters and conditions that lead to widespread chaotic advection. In the course of flow evolution an initially designated interface of any petroleum patch may become extremely convoluted, and it appears difficult to follow pattern structures in full detail. In this study we use the special algorithm developed in our previous paper [7], in which extra contour (surface) points are automatically added when needed. Attention will also be given to a quantitative estimation of the transport quality based upon measures first suggested by Gibbs [8] and Welander [9], such as coarse-grained density. Such coarse-grained representations over an investigation area, show a "residence place" for the pollutant material at any instant. If initially it occupies any box *ij* of the whole marine zone, divided by square boxes, nonzero elements of the exchange matrix show places where pollutant particles go.

EVALUATION OF CHAOTIC TRANSPORT

In the present study of the petroleum patch transport in tidal areas we use Zimmerman's kinematical model [3, 6] which is based upon a superposition of a tidal and a residual current flow field. The kinematical model describes a motion of a mathematical point that moves at each instant with the velocity corresponding to its instant position. The petroleum particle is supposed to be inertialess, not subjected to diffusion or interfacial tension. The residual time independent current field is an infinite sequence of clockwise and anticlockwise rotating eddies. Streamlines of the residual velocity field divide the whole area into square cells of equal area, with elliptic points in the centers and hyperbolic points in the corners of cells. The tidal field plays a role of a pertubation of the Hamiltonian system (the stream function is the Hamiltonian for the residual flow). In general this pertubation leads to chaotic dynamics and could be studied in different ways, for example, by means of Poincaré sections. However, in pollutant spreading problems, we are interested in the short term history of pollutant transport. Poincaré sections present the history of motion of points in some area during a long time interval, say, during a thousand periods of a tidal flow. (For the problem of petroleum patch transport this corresponds to the history of one point during almost one and a half year.) On the contrary, ecological considerations demand that disastrous spread of pollution has to be stopped in days or weeks. Therefore, we need to know which part of the Eulerian space will be polluted in a short time and, more importantly, how much petroleum will leak to some specific part of a sea. The orbit expansion method [6], developed for a quantification of the chaotic transport and exploited an assumption that the contributions of tidal and residual currents are of different orders (the tidal is much stronger), does not give answers to those questions. In our case, it is important to know not the mixing region (where presumably mixing is instantaneous) obtained by the long time tool – Poincaré sections, -- or the rate of material exchange (which could be high in a very narrow domain), but how uniformly this mixing region is distributed over the whole area during a specific finite interval of time. We suggest a different approach for an estimation and quantification of pollutant transport.

Here we briefly present a methodology for the quantification of the chaotic transport [7] based on the statistical quantities such as a coarse-grained density. First step is to present a petroleum patch, for example, as a circular blob continuously occupying some part in marine or coastal zone. Then we use a contour tracking algorithm that conserves both area and topological properties (connectedness and non-self-intersection) to find the blob's boundary in Eulerian space at any moment of time. In our algorithm the key idea is the use of a non-uniform distribution of points at the initial contour to present this interface in such a way that (i) the distance between neighbouring points remains between some chosen values (for that points are added when the distance becomes long enough and points are removed when it becomes too short) and (ii) the angle between any neighbouring straight lines is larger than some prescribed value. The principal advantage of our algorithm is that area preservation of the blob enclosed by the contour is guaranteed even after high stretching and complicated folding. Knowing the position of the contour line (the boundary of the petroleum patch) we can construct an Eulerian description of the mixing process, giving an opportunity to quantify mixing at any moment of time.

The next step is dividing the whole area of investigating marine zone S into N square boxes of a side size δ with an area $s = \delta^2$ each, then the marine zone area can be written as $S = Ns$. The conservation of petroleum area enclosed by the contour line permits us to introduce a probability function for the petroleum distribution inside a box with number k as proportional to value of area P_k occupied by petroleum in this box. A ratio of P_k and s can be called a probability density or a coarse-grained density. Figure shows the highly ramified petroleum patch (a) and the coarse-grained representations (with (b) δ=1 and (c) δ=0.3) of the mixing pattern after 9 periods of the tidal flow in the zone with 6 cells of the residual current in x and in y directions. The lightest boxes correspond to areas without petroleum.



At any instant the coarse-grained representation of the petroleum patch distribution clearly demonstrates which boxes are empty, and which have the petroleum material and how much. It may also be used to show the exchange of material inside boxes, if we study this process and construct a special exchange matrix for the tidal flow in the area under consideration. Then using this matrix we can predict transport of petroleum from any place (any box) in the area to an arbitrary location in the tidal sea and determine the time when it happens.

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