ENHANCED MIXING BY VORTICES

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Summary The mixing of a passive scalar blob by the simple stirring protocol realized in the deformation field of an axisymmetric vortex is investigated. The complete transient concentration field exhibits an accelerated diffusion of the scalar, due to its stretching in a spiral surrounding the vortex. The concentration histogram of the scalar presents an unexpected cusp at early stages. The long time properties of the histogram and the scalar dissipation rate are also examined.

INTRODUCTION

A central question in scalar mixing is the description of the histogram or probability distribution function (PDF) $P(c)$ of the concentration level c of a substance being mixed. In the presence of a velocity field, the diffusion is altered by the stirring of the substrate and is usually enhanced, as for example in a straining field [1] or in a vortex [2,3]. However, experimental and theoretical works addressing the structure and origin of the scalar PDF are scarce. Based on a spatially and temporally resolved experiment, we study the mixing chronology of a blob of dye embedded in the displacement field of a Lamb-Oseen vortex, from the initial segregation of the blob to the state where it is almost completely diluted in the medium [4].

ACCELERATED DIFFUSION IN A VORTEX

A blob of dye (dark patch on Fig. 1) with initial size s_0, concentration c_0 and diffusivity D is placed far from the vortex center of initial size a_0 . The vortex can therefore be considered as a point vortex, with azimuthal velocity $v_\theta=\Gamma/2\pi r$ (Γ=14 cm^2/s being its circulation). The passive blob rolls up around the vortex in a spiral, and its length increases linearly in time. By a local analysis in a frame of reference aligned with the spiral, the search for $c(r,t)$ is reduced to a simple problem of diffusion in a stretching field whose amplitude depends on the radius from the vortex center, and which can be solved by a classical change of variable [1], leading to an exact solution of the concentration field [4]. A simple expression for the maximal concentration c_M at each radius follows:

$$c_M(r,t) = c_0 \;\; erf\left[\frac{1}{4}\left(\frac{Dt}{s_0^2}+\frac{D\Gamma^2 t^3}{3\pi^2 r^4 s_0^2}\right)^{-1/2}\right]$$

The maximal concentration has two different behaviours whether the fluid particles in the spiral have lived less or more than a mixing time t_s, obtained when the term inside the error function equals unity:

$$t_s(r) = \frac{r^2}{\Gamma}\left(\frac{3\pi^2}{16}\right)^{1/3}\left(\frac{s_0}{r}\right)^{2/3}\left(\frac{\Gamma}{D}\right)^{1/3}$$

Before the mixing time, the striation thickness is larger than the diffusion length and the maximal concentration c_M is constant. After the mixing time, enhanced diffusion is observed, for which c_M decreases as $t^{-3/2}$ instead of $t^{-1/2}$. This is due to the stretching of the spiral as t added to a normal diffusion in the contracting coordinates. This prediction contains no fitting constant and is plotted on Fig. 2a for a fixed radial location together with the experimental measurements.

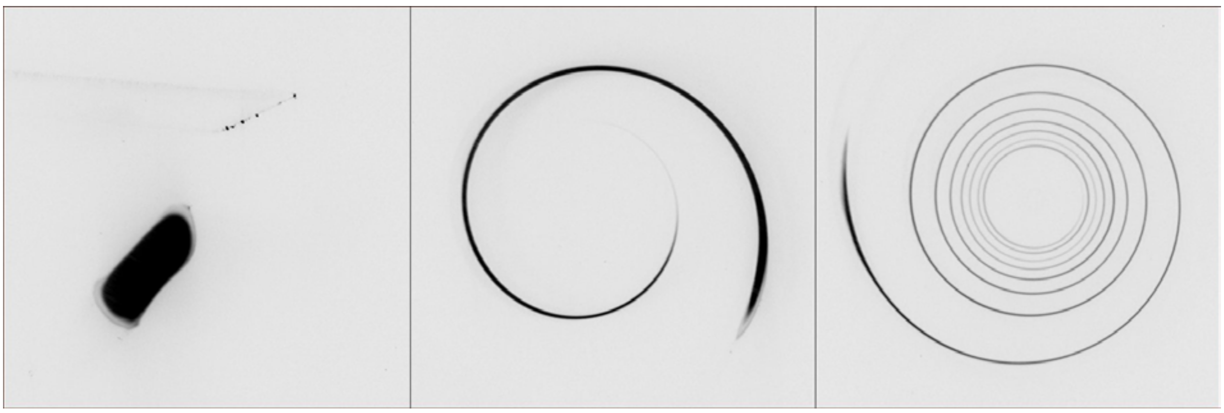

Fig. 1: Roll-up of a blob of fluorescent dye in a point vortex at t = 0, 2 and 10 s. The field of view is 4.8 cm wide.

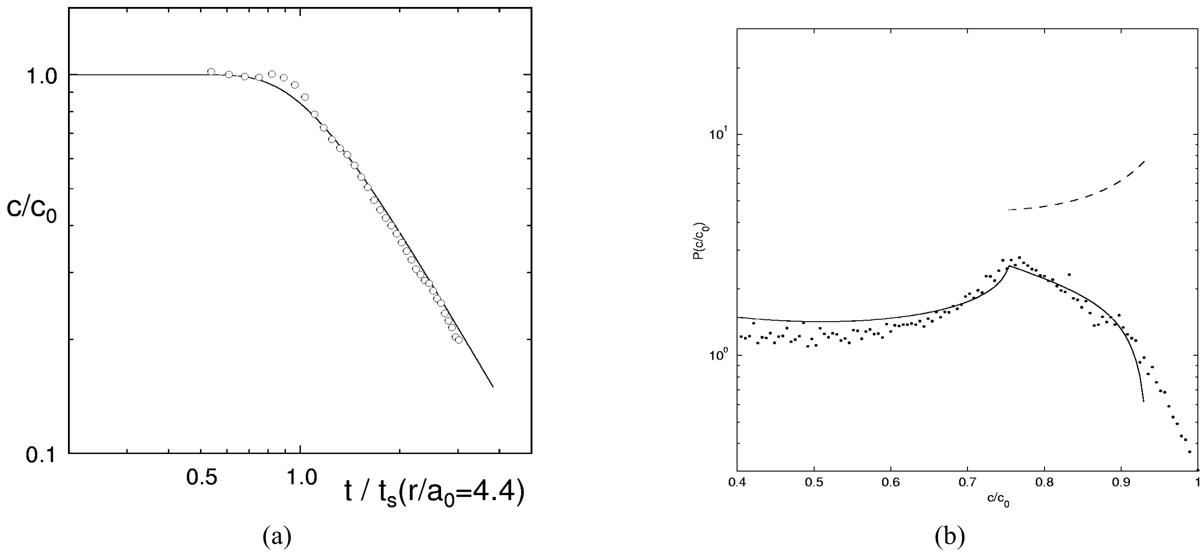

Fig. 2: (a) Maximal concentration of the dye obtained experimentally (symbols) and theoretically (solid line) at a radius $r=4.4\,a_0$. (b) Probability density function of the dye concentration at t =13 s. Theoretical prediction is made for the global scalar field (solid line) and for the maximum dye concentration of the spiral (dashed line).

HISTOGRAM OF THE PASSIVE SCALAR

After the mixing time, the transverse concentration profile across the spiral arms is a Gaussian function parametrized by c_M. By extracting the shape of the iso-concentration lines, it is possible to derive an analytical solution of the PDF of the concentration $P(c)$ at any time [4]. An example is shown on Fig. 2(b) at a given intermediate stage. The PDF of the maximal concentration c_M is also shown as a dashed line on Fig. 2(b). The global PDF can be understood as the sum of U shaped histograms based on each maximal concentration. This superposition introduces a cusp at the minimum of the maximal concentration, obtained for the minimum radius of the blob of dye. When this radius is small, or at late stages, the cusp vanishes and the PDF tends to a pure power law

$$P(c) \sim c^{-3/2}$$

at late stages.

SCALAR DISSIPATION RATE

The exact solution of the concentration field also provides the scalar dissipation rate $\chi=-(d/dt)\langle c^2\rangle$. If all the elements of the spiral have not reached the mixing time, χ scales as $t^{1/2}$, as shown by Flohr and Vassilicos [2]. However, if all the elements of the spiral have reached the mixing time, *i.e.* when there is no dye with the initial concentration, the dissipation rate scales as $t^{-5/2}$.

CONCLUSIONS

The simple stirring protocol considered here allows to reach an exact solution of the concentration field and of the PDF of the concentration, in close agreement with the experimental data. Useful scalings are derived at late stages for the PDF of concentration and for the scalar dissipation rate.
The evolution of the concentration field during the merging of two independent vortices will be analyzed as well.

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