Ternary Alloy Convection in Mushy Layers: Extended Summary

D.M. Anderson
Department of Mathematical Sciences
George Mason University
Fairfax, VA 22030, USA

T.P. Schulze
Department of Mathematics
University of Tennessee
Knoxville, TN 37996, USA

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Much has been learned over the past several decades on convection and solidification in binary alloy mushy layers (for reviews see Worster[1, 2]). However, many industrial applications and geological processes involve multicomponent alloys. We are motivated by this and the success of the binary alloy models to extend the modeling efforts to ternary alloy systems.

The presence of multiple chemical species complicates even the equilibrium characterization of the known material phases. The complexities of the thermochemistry for common industrial multi-component superalloys leaves little room for mathematical analyses of these systems. Typically, one must resort almost immediately to numerical solution of the models to make headway. Recently, however, a window of opportunity has been identified by Aitta *et al.*[3, 4] who performed an experimental investigation on the solidification of the aqueous solution of potassium nitrate ($KN0_3$) and sodium nitrate ($NaN0_3$). This ternary alloy system has a relatively simple equilibrium thermochemistry, at least in comparison to its metallurgical counterparts, that is characterized well enough to merit theoretical investigation. In particular, it was found that a ternary eutectic phase diagram with complete solid immiscibility was suitable to describe the phases formed during solidification. Their experimental system has proved not only feasible for mathematical investigation but also of great scientific interest, as a number of interesting phenomena have been identified and are as yet not completely understood.

In the experiments of Aitta *et al.*, H_2O–$KN0_3$–$NaN0_3$ was solidified from below by a cold, planar boundary at $z = 0$. The operating state was in a diffusion-controlled regime; convection, including that due to thermal and solutal buoyancy, was absent. Their experiments showed the formation of two distinguishable mushy layers between a liquid layer above and a ternary eutectic solid layer below. Due to the complexities of the double mushy layer geometry, this system has convecting regimes about which very little is known.

We describe here recent work on a diffusion-controlled model of the solidification of ternary alloys in mushy layers[5]. In particular, we identify one-dimensional similarity solutions describing the temperature and composition profiles through the layers, as well as the solid fraction profiles through the mushy layers for non-convecting scenarios. Detailed investigations of how the growth characteristics, such as the layer thicknesses, depend on the system control parameters are given. These results compare well with the experimental work of Aitta *et al.*

The double mushy layer geometry in the ternary alloy system gives rise to convective scenarios that are not present during binary alloy solidification. As described by Aitta *et al.*[4] and Anderson[5] the ternary system can develop two mushy layers in which compositional convection occurs in (i) neither mushy layer, (ii) the primary layer only, (iii) the secondary layer only, or (iv) both layers. These guidelines offer clues to the likely convection in the ternary system but they do not address how convection in one layer may influence that in another layer. We investigate these issues in more detail by identifying convecting solutions in the double mushy layer geometry and analyzing their stability properties.

References

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