MECHANISMS OF PARTICLE DEPOSITION IN A TURBULENT FLOW

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Summary A study of the mechanisms governing deposition was carried out by using a stochastic model for the turbulence seen by the particles.

The understanding of dispersion and deposition of particles in a turbulent flow has been advanced by studies of the behavior of a point source in a direct numerical simulation of flow in a channel. This work has been limited because of the large amount of computer time that is required. An alternate approach is to represent the turbulence seen by the particles with a stochastic model. This paper describes studies that use a modified Langevin equation to do this. This allowed a consideration of a wide range of variables, very large computation times and a larger Reynolds number than has been used in DNS studies. Results are presented on the mechanisms of deposition under conditions that gravitational effects are not important.

The system considered is an idealized annular flow in a horizontal channel. Spherical solid particles are injected at a velocity V_i from sources located at the bottom and top walls. Deposition occurs when particles are a distance of one radius from the wall. The concentration field is calculated by representing the two walls as being a collection of sources. A stationary state is reached when the rates at which the particles are injected into the field from the bottom and top walls equal the rates of deposition, $R_{AB} = R_{DB}$ and $R_{AT} = R_{DT}$. This paper considers results only for $g_i = 0$, so they are the same as for flow in a vertical channel if lift forces are negligible.

The main theoretical problem is to describe the behavior of a single source. Dilute flows with particles that are much heavier than the gas are considered. Thus, the lift force, inter-particle collisions and the influence of particles on the gas flow are ignored. The location and velocity of a particle are defined with the following equations:

$$\frac{dx_i}{dt} = V_i \tag{1}$$

$$\frac{dV_i}{dt} = -\frac{3\rho_f C_D}{4 d_p \rho_p} |\boldsymbol{V} - \boldsymbol{U}| (V_i - U_i) + g_i \tag{2}$$

where x_i is the displacement of the particle, V_i is the velocity of the particle, U_i is the gas velocity seen by the particle, t is the time, ρ_p is the density of the particle, ρ_f is the density of the gas, and g_i is a component of the acceleration of gravity.

$$C_D = \frac{24}{Re_p}\left(1 + 0.15 Re_p^{0.687}\right) \tag{3}$$

where the particle Reynolds number, Re_p, is defined with d_p and the magnitude of the relative velocity $|\boldsymbol{U} - \boldsymbol{V}|$. The inertial time constant of a particle is defined as

$$\tau_p^+ = \frac{4 d_p^+ (\rho_p / \rho_f)}{3 C_D \left|\boldsymbol{V}^+ - \boldsymbol{U}^+\right|} \tag{4}$$

For Stokes law resistance

$$\tau_p^+ = \frac{d_p^{+2} (\rho_p / \rho_f)}{18} \tag{5}$$

The + superscript indicates that the quantities are made dimensionless using the friction velocity, v^*, and the kinematic viscosity, ν.

The mean fluid velocity seen by the particles, $\overline{U}_i$, is given by measurements. The fluctuations, u_1, u_2, u_3 in the x_1, x_2, and x_3 directions are calculated either from a DNS or from the stochastic representation. Results obtained using these two approaches were compared at $Re_\tau = Hv^* \rho_f / \mu = 150$ (where H is the half-height of the channel) to insure the accuracy of the method. Calculations of the deposition constants are presented in this paper for $Re_\tau = 590$.

The modified Langevin equation is used to calculate the change of the fluid velocity fluctuation seen by a solid particle, du_i, over a time interval dt:

$$d\left(\frac{u_i}{\sigma_i}\right) = -\frac{u_i}{\sigma_i \tau_i} dt + \overline{d\mu_i} + d\mu_i' \tag{6}$$

where u_i is the fluctuating component of the fluid velocity seen by the particle, σ_i is the Eulerian root-mean-square value of the velocity fluctuations, τ_i is the Lagrangian time constant. The forcing function is assumed to be jointly Gaussian (Mito and Hanratty, 2002). We follow the simple approach , described by Mito and Hanratty (2003), that uses time constants characterizing the dispersion of fluid particles to define τ_i.

Calculations were done at $Re_\tau = 590$ with $V_{i2}^+ = 1$, $V_{i1}^+ = 15$. Two types of studies were done. In one of these the particle diameter was kept constant at $d_p^+ = 0.368$ and the Stokesian τ_p^+ (Eq. 5) was varied between 3 and 2.5×10^4. This implied that ρ_p / ρ_f was varied from 1.33×10^2 to 3.32×10^6. In the other, ρ_p / ρ_f was kept constant at 10^3 and the Stokesian time constant was varied from 1 to 10^5. These required a variation of $d_p^+ = 0.134$ to $d_p^+ = 42.4$.

The mechanism of deposition can be illustrated by considering the velocity with which particles deposit on the wall, V_d^+. The cumulative contribution to the flux by particles with velocities V_d^+ and less, normalized by the rate of deposition, are plotted in Fig. 1 for $d_p^+ = 0.368$. The parameter τ_p^+ is the dimensionless Stokes inertial time constant defined by Eq. (5). For $\tau_p^+ = 1, 3, 5$ most or all of the particles deposit with velocities of $10^{-4} < V_d^+ < 10^{-3}$, which is of the same magnitude as the fluid turbulence at a distance from the wall equal to the particle radius. These results and measurements of the concentration profiles show that the particles accumulate in a region very close to the wall (say $x_2^+ < 2$), where they have a jitter transmitted by the fluid turbulence. This jitter causes a small fraction to strike the wall and deposit.

For $\tau_p^+ = 10, 20, 40$ the main contributions are made by particles depositing with a much larger velocities characteristic of the fluid turbulence farther away from the wall. Because of their larger inertia some of the particles are pictured to disengage from the turbulence and to proceed to the wall by a "free-flight." These "free-flights" by particles outside the viscous sublayer are the dominant mechanism for deposition even though a high concentration of trapped particles can exist close to the wall. As τ_p^+ increases the particles, on average, start their free-flights from locations with increasing distances from the wall. Thus the average deposition velocity, $\overline{V}_d^+$, increases with increasing τ_p^+. For large enough τ_p^+ they start free-flights outside the viscous wall layer, say $x_2^+ > 40$. This observation and the finding that concentration profiles tend to be uniform at large τ_p^+ lead to the recent proposal (Hay, Liu, Hanratty, 1996) that the deposition constant defined as $k_{DB} = R_D / C_B$ is given by

$$k_{DB} = \frac{\sigma_p}{\sqrt{2\pi}} \tag{7}$$

where σ_p is the root-mean square of the wall-normal velocity fluctuations of particles. A particle turbulence with a Gaussian distribution is assumed.

Calculations of k_{DB}^+ and of $k_{DW}^+ = R_D / C_W v^*$ are presented in Fig. 2 for a range of τ_p^+ (defined by Eq. 5) of 3 – 25000. The curve with dots and dashes represents Eq. (7). Brownian motion is represented by a Schmidt number given by $Sc = 3\pi\nu d_p \mu / kT$, where ν is the kinematic viscosity, d_p is the particle diameter, μ is the viscosity, k is the Boltzmann constant and T is the absolute temperature. The mass transfer coefficient, K, for large Sc has been given by Shaw and Hanratty (1977) as

$$K^+ = 0.0889 Sc^{-0.704} \tag{8}$$

This is plotted in the bottom corner for $Sc = 2.2 \times 10^6 - 5.0 \times 10^6$, which would represent flow of air at atmospheric pressure with a friction velocity of $v^* = 0.6$ m/s. Several regimes can be identified: (1) Deposition is controlled by Brownian motion, $\tau_p^+ \leq 1$; (2) Deposition by particles trapped in a layer close to the wall, $1 \leq \tau_p^+ \leq 5$; (3) Free-flight from different locations in the viscous wall layer, $10 \leq \tau_p^+ \leq 40$; (4) Free-flight from regions outside the viscous wall layer, $40 \leq \tau_p^+ \leq 2000$; (5) Free-flight from one wall to another, $\tau_p^+ > 2000$. Annular flows are represented by regimes 4, 5.

References

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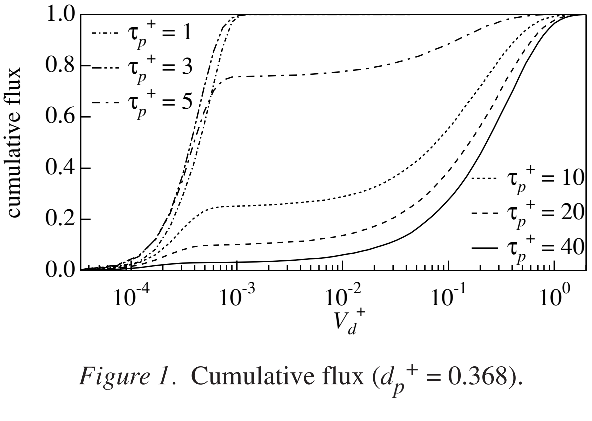

Figure 1. Cumulative flux ($d_p^+ = 0.368$).

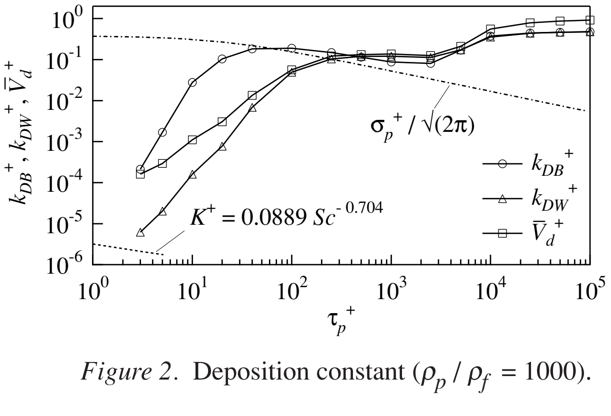

Figure 2. Deposition constant ($\rho_p / \rho_f = 1000$).