ASPECTS OF THE LAMINAR-TURBULENT TRANSITION IN AXISYMMETRIC BOUNDARY LAYERS

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Summary

The boundary layers over axisymmetric bodies have been studied far less than those over two-dimensional surfaces. Our main obective is to understand how the laminar-turbulent transition process in a boundary layer over an axisymmetric body is different from that over a two-dimensional surface. We study the primary (linear) instability, the secondary instability, and the transition zone, and find that all these are qualitatively different from 2D boundary layers. It is shown that transverse curvarture has a significant stabilizing effect on the primary stability. Consistent with the recent findings of Tutty *et al* [1], we see that three-dimensional modes can go unstable first, whereas over a 2D surface 2D modes are most unstable (Squire's theorem[2]), Interestingly, an opposing effect of curvature is seen on secondary instability behaviour: competing primary modes produce a rich variety in secondary disturbance growth, indicating early entry into the nonlinear domain. Early stages of the transition zone, where turbulent spots grow as they convect, are similar to 2D flow, while due to the spots wrapping around the body, transition proceeds slower in the later stages.

INTRODUCTION

The laminar to turbulent transition in a two-dimensional boundary layer has been a subject of great interest for several decades, but the boundary layer created by axial flow past an axisymmetric body, such as in underwater applications, has received far less attention. We find the laminar instability and transition process in axisymmetric boundary layers to be qualitatively different from its two-dimensional counterpart at every stage, and there are aspects of mathematical interst. We study secondary and algebraic disturbance growth to determine whether, and which of, these mechanisms are dominant in axisymmetric boundary layers.

SUMMARY OF MAIN RESULTS

A summary of our findings is presented schematically in figure 1. Transverse curvature has the effect of stabilizing the primary (linear) mode, so the first critical Reynolds number is higher. A totally new route to transition is however possible due to the geometry since Squire's theorem [2], stating that 2D modes are the most unstable primary (linear) modes, may now be violated. (Due to the transverse curvature, the resulting stability equations cannot be transformed in the manner proposed by Squire.) Our linear stability studies find that, at various levels of curvature, three dimensional modes of azimuthal wavenumber $n = 1$ or 2 are indeed the least stable (figure 2a). These results are consistent with the findings of Tutty *et al* [1].

A secondary instability analysis is carried out in the standard manner following Herbert[3]. Secondary disturbance growth is much more complex than in 2D. Since several primary modes are nearly as stable as each other, a variety of secondary modes are triggered at a given Reynolds number. Nonlinear interactions among these modes are therefore likely to bring the transition Reynolds number forward, as depicted in figure 1. The growth of secondary disturbances for a few example modes is shown in figure 2b, a detailed analysis and comparison with 2D will be presented at the conference. As in the flat plate boundary layer the sub-harmonic modes are found to be more dominant than the harmonic.

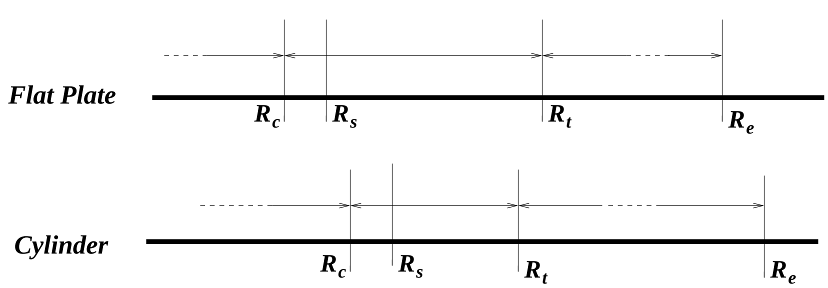

Figure 1. Schematic representation of the differences in onset of different stages of the transition process in 2D and axisymmetric boundary layers. R_c is the Reynolds number of linear instability, R_s is the onset of secondary disturbance growth, R_t is the onset of transition to turbulence, and R_e shows the end of the transition zone, where the flow is fully turbulent.

We have carried out stochastic simulations inspired by a cellular automaton approach of the birth and downstream propagation and growth of turbulent spots in the transition zone of an axisymmetric boundary layer, details will be presented at the conference. The intermittency γ, the fraction of the time that the flow is turbulent at a given streamwise location x

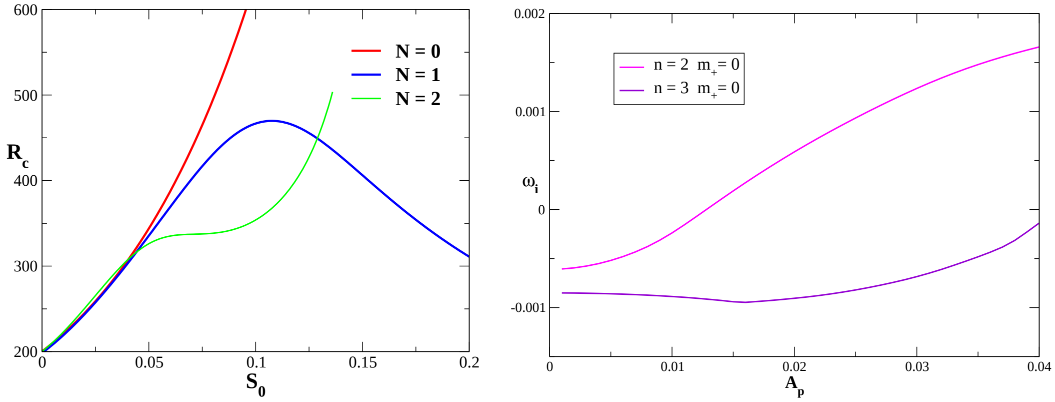

Figure 2. (a) Variation of critical Reynolds number with curvature, S for different modes,[4]. Growth of secondary waves with different A_p at Reynolds number 300, primary wavenumber $(\alpha) = 0.10$, $S_0 = 0.05$ and n and m are the number of primary and secondary waves encircling the body respectively. A_p is the amplitude of the primary disturbance.

is shown in figure 3 a. The quantity c is the circumference of the body. In the initial region, transition proceeds exactly as it would in 2D flow. This is because the spots are too small to "see" the body. When spots wrap themselves around the cylinder, however, there is no further room for lateral growth, and transition proceeds much more slowly after this. The burst rate B shown in figure 3 b is another indication of the differences in the transition process. Here, B is a measure of the rate at which the flow switches from laminar to turbulent.

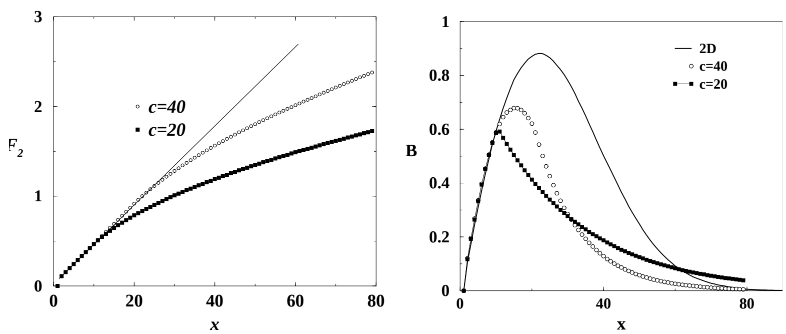

Figure 3. a. The intermitteny factor $F_2 \equiv \sqrt{-(log(1-\gamma(x))}$ Vs. x, for different c. The straight line is the 2D result. b. The burst rate in the transition zone.

Acknowledgement

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