A combined numerical and asymptotic approach to boundary layer receptivity problems

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SUMMARY: With the aim of predicting the transition point on a general body given the free-stream conditions, we devised a method using asymptotic receptivity analysis as an upstream boundary condition for numerical stability analysis.

For small free-stream disturbances, the problem of predicting transition can be split into two parts, the receptivity part and the stability part. Receptivity is concerned with how the free-stream disturbances give rise to disturbances in the boundary layer and typically arises when the boundary layer changes over short streamwise scales, such as near the leading edge of a body.

For large Reynolds number, Re, the neutral stability point is $O(\epsilon^{-2})$ downstream of the leading edge, where $\epsilon = Re^{-\frac{1}{6}}$, and its position is independent of the exact form of the free-stream disturbance in the receptivity region. The amplitude of the boundary layer mode which eventually becomes unstable is determined by a receptivity region. Downstream of the neutral stability point, disturbances grow, leading eventually to nonlinear breakdown and transition to turbulence. Thus the amplitude of the unstable mode at the neutral stability point is crucial in determining the position of transition.

Three approaches to understanding the transition process are available.

(i) **Experiments**: Experimentalists, e.g. Saric *et al.* (1995), typically measure the transition point and the amplitude of the Tollmien-Schlichting (T-S) wave downstream of the neutral stability point.

(ii) **Numerics**: Numerical Navier-Stokes (N-S) solvers have been used to solve the receptivity/stability problem simultaneously (Wanderley and Corke, 2001). However these solvers only work for moderately large Reynolds numbers, obtaining solutions is very time consuming, and identifying the amplitude of the unsteady modes which eventually become unstable is difficult.

(iii) **Asymptotics**: At large Reynolds numbers, the receptivity process close to the leading edge is analysed separately and then matched to a triple-deck asymptotic analysis of the Orr-Sommerfeld region (Goldstein, 1983). The asymptotic structure is summarised in figure 1. The advantage of such an approach is that scaling on Reynolds number and dependence on the exact form of the free-stream disturbance is directly determined. However, even for a semi-infinite flat plate, the asymptotic analysis of the Orr-Sommerfeld region is so intricate that comparing wave amplitudes at the neutral stability point with experiments is impractical.

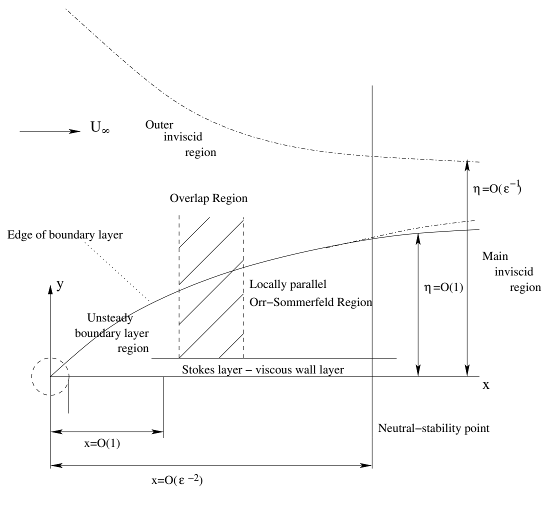
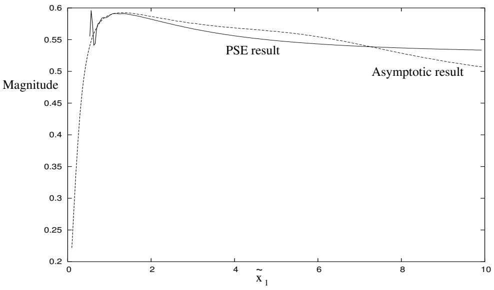
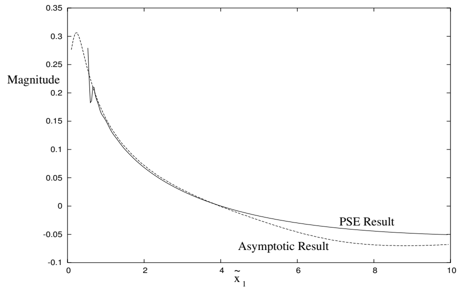


Figure 1: Figure showing the regions on a flat plate



(i)



(ii)

Figure 2: Comparison of numerical solution of PSE and asymptotic results of *(i)* the real and *(ii)* imaginary parts of the wavenumber , κ, as a function of downstream distance for $\epsilon = 0.1$

In this paper we consider an asymptotic analysis of the receptivity region and then use the far-downstream limiting form as an initial condition for a numerical solution marching downstream using the Parabolized Stability Equation (PSE) (Bertolotti *et al.*, 1992). The PSE method reduces computation time greatly compared to full N-S solvers. The advantage of the combined approach compared to the fully asymptotic treatment is that bodies with curved surfaces can easily be considered. However, before we can extend this method to more general bodies, we first present comparisons with the semi-infinite flat plate analysis.

Figure 2 shows the comparison between the numerical solution to the PSE and Goldstein's asymptotic solution for the wavenumber, κ, as a function of distance downstream, for $\epsilon = 0.1$. We use as our initial condition a composite solution constructed from the three decks of the eigensolution determined by the receptivity analysis. Downstream of the starting point of the PSE calculations there is good agreement between numerical results and the asymptotic predictions, at least as far as the neutral stability point, located at $\tilde{x}_1 \approx 3.65$. Further downstream agreement is not as good, probably due to non-uniformity in the small-ϵ expansion of κ. However, this is not significant since it is the wave amplitude at the neutral stability point which is of prime importance in the current work. Reducing ϵ improves the agreement, although it has not been possible to demonstrate that all orders in the asymptotic expansion of κ are correctly determined. The rapid variation in κ in the early stages of the PSE calculation are due to the adjustment in the mode shape from the composite solution based on the small asymptotic theory. In the example shown, PSE calculations were started at $\tilde{x}_1 = 0.5$. Calculations were repeated starting from other values of $\tilde{x}_1$ and the behaviour further downstream showed similar agreement with asymptotic results.

We also compared the wave shape from the PSE analysis with the triple deck asymptotic form from Goldstein's Orr-Sommerfeld analysis. The results were very encouraging, although no results are printed here.

CONCLUSIONS

The results shown confirm the PSE formulation has been implemented correctly in the numerical codes and the upstream boundary conditions imposed are consistent. Results will be presented for bodies with non-zero mean pressure gradients and comparisons made with N-S results and experimental data.

References

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