NUMERICAL STUDY OF THE FLOW IN A FINITE CYLINDER DRIVEN BY A ROTATING MAGNETIC FIELD

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Summary We present direct numerical simulations (DNS) of the flow in a finite cylinder driven by a rotating magnetic field. The objective of this study is to identify the dominating structures and mechanisms in the transitional and early turbulent flow regimes. The DNS cover a range up to $10\,Ta_{\mathrm{c}}$, where Ta denotes the magnetical Taylor number. The main result of the study is the insight in the formation, evolution and finally dissipation of Taylor-Görtler-like vortices that clearly dominate the turbulence physics and provide an efficient mixing mechanism in this flow.

INTRODUCTION

Electromagnetic stirring provides an appealing method for non-intrusive flow control in metallurgy and crystal growth. In this study we focus on the flow in a finite cylinder driven by a rotating magnetic field. This topic has been subject to extensive research since the pioneering work of Moffat [1]. A review of ealier work is given, e.g. in [2]. Concerning stability of the laminar flow significant progress has been made only recently, e.g. [3,4]. Our knowledge of the turbulent regime, however, relies on quite few experimental data and semi-empiric theories [5].

NUMERICAL APPROACH

Employing the rigid-body and low-frequency approximations the mathematical model reduces to the Navier-Stokes equations with a priori known Lorentz force. A three-dimensional, second-order finite-element method combined with Adams-Bashforth time integration is used for discretization. The direct numerical simulation were performed in parallel on an SGI multiprocessor system. The computations cover a range up to $10\,Ta_{\mathrm{c}}$, where Ta denotes the magnetical Taylor number.

RESULTS

In agreement with the semi-empiric model of Davidson [5] and the little available experimental data, the mean flow is characterized by a homogeneously rotating core. The top and bottom Bödewadt layers drive a weak secondary flow which, however, plays an important role in balancing the Lorentz and friction forces, see Fig. 1.

The most striking result of the study is the insight in the formation, evolution and finally dissipation of Taylor-Görtler-like vortices that clearly dominate the turbulence physics and provide an efficient mixing mechanism in this flow. Typical large structures identified by the second invariant of the fluctuation velocity gradient are shown in presented in Fig. 2. The coloring indicates the orientation of the rotation axis. As might be expected, pairs of Taylor-Görtler-like vortices represent the preferred structure at moderate supercritical flow. Our analysis shows that these vortices survive for several rotations of the mean flow. Near top and bottom they are drawn into the Bödewadt layers where they are finally dissipated. At higher Taylor numbers, the vortices become more unstable and irregular but still provide the dominating turbulence mechanism. In addition to these vortex structures, large scale flow variations in azimuthal direction could be identified. However, as is evident from frequency and wave number spectra, these fluctuations do not contain any harmonic components even in near-critical flow.

CONCLUSION

The transient and early turbulent flow driven by a rotating magnetic field was investigated in detail by numerical computation. Consistently to the model by Davidson, the top and bottom Bödewadt layers drive a weak secondary flow, forcing fluid from the core region to pass the viscous wall layers. Furthermore, the DNS revealed that the turbulent flow is at least up to 10 Ta_{c} dominated by long-living Taylor-Görtler vortices.

References

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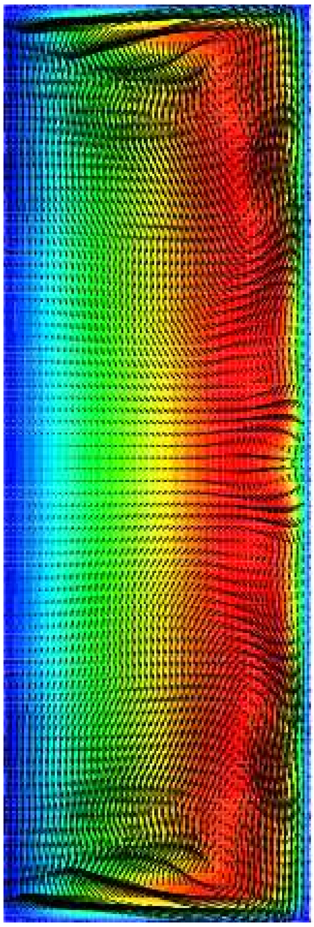

Figure 1. Mean velocity distribution at $Ta = 10^5$

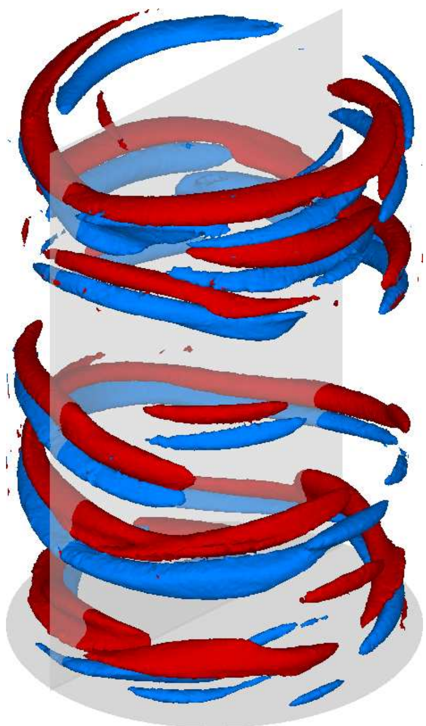

Figure 2. Instantaneuous vortex structures at $Ta = 10^5$